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ON THE FRATTINI SUBGROUPS OF GENERALIZED FREE PRODUCTS

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1. **Introduction.** In [**1**] Higman and Neumann answered a question of N. Ito whether free products of groups necessarily have maximal subgroups by showing that the Frattini subgroup of any free product of groups is trivial. From this it follows that free products of groups always have maximal subgroups. In the same paper Higman and Neumann raised the questions whether generalized free products of groups necessarily have maximal subgroups and whether the Frattini subgroup of a generalized free product is always contained in the amalgamated subgroup. No substantial result in the way of answering these questions had been obtained until Whittemore [**5**] showed that if $G=(A * B)_H$ is the generalized free product of A and B amalgamating H and if there exists $x \in G$ such that $H \cap H^x=1$ then the Frattini subgroup $\Phi(G)$ of G is trivial. Using this result Whittemore, in the same paper, showed that the Frattini subgroup of the generalized free products of finitely many free groups amalgamating a cyclic subgroup is trivial. This result was improved by Tang [**4**]: If $G=(A * B)_H$, where A and B are free and H is finitely generated such that at least one of $[A:H]$ and $[B:H]$ is infinite then $\Phi(G)=1$. In the same paper it was also shown that if $G=(A * B)_H$ and $\Phi(G) \cap H=1$ then $\Phi(G)=1$. We can now greatly improve this result.

2. **Main Theorem.** Notations and terminology will be the same as in [**4**].

THEOREM 1. *Let $G=(A * B)_H$. If G contains a nontrivial normal subgroup N such that $N \cap H=1$ then $\Phi(G)$ is contained in the maximal G-normal subgroup contained in H.*

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