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BRANCHED AND FOLDED PARAMETRIZATIONS OF THE SPHERE

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0. This study is addressed to the following genre of topological problems. Let Π be a subset of a manifold W and $\varphi:\Sigma\to\Pi$ be a parametrization of Π by a manifold collection Σ. We seek a factorization $\Sigma \to^{i} M \to^{F} W$, $\varphi=F\circ i$, where i is an inclusion of Σ in a manifold M of the same dimension as W and F is a map in a certain class, such that the invariants of (W, Π, φ) in some reasonable sense determine (M, F, i) up to topological equivalence. For instance, let Π be a closed, but not necessarily simply closed polygon in the complex plane W, Σ the extended real line and F a Schwarz-Christoffel transformation of the Gaussian upper half plane M, such that the image $[\varphi]$ of $\varphi=F|\Sigma$ coincides with Π. Necessary and sufficient conditions for Π to bound a conformal, or more generally, a holomorphic image of a disc were first given by Titus [**11**]. In view of the Stoïlow-Whyburn [**16**] theory, it proved more convenient to use light open maps F such that φ is a regular parametrization of a smooth, closed curve Π. If the curve lies in general position, the conditions can be expressed in terms of the Whitney [**14**]-Titus [**10**] intersection sequence, which is a combinatorial structure on the set of signed self-intersection points $X(\varphi)$ of Π. In the last decade considerable progress has been made in the direction of relaxing the specialized aspects of the Picard-Loewner problem solved by Titus. We present here some current work, the precise formulation of some technical definitions and proofs have or will appear elsewhere.

1. Let M denote a smooth, compact oriented surface, possibly with boundary ∂M, and W a smooth, oriented surface without boundary, but with base point ∞. We admit smooth maps $F: M\to W$ which in the vicinity of a point $m\in M$ is locally smoothly equivalent to one of the following canonical plane maps near the origin:

$m\in B$ is a *branch point* ($w=(x+iy)^{\nu}$, $\nu>1$) of *valence* ν,
$m\in C$ is a *fold point* ($w=x^2+iy$),
$m\in K$ is a *cusp point* ($w=x^3-xy+iy$),
$m\in P=F^{-1}(\infty)$ is a simple *pole point* ($w=(x+iy)^{-1}$),
$m\in J=\partial M$ is a *border point* ($w=x+iy$, $y\geqq 0$),
$m\in M_0$ is a *regular point* ($w=x+iy$).

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