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AUTOMORPHISMS OF THE LATTICE OF RECURSIVELY ENUMERABLE SETS

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Let $\mathscr{E}$ denote the lattice of recursively enumerable (r.e.) sets under inclusion, and let $\mathscr{E}^*$ denote the quotient lattice of $\mathscr{E}$ modulo the ideal $\mathscr{F}$ of finite sets. For $A \in \mathscr{E}$ let A^* denote the equivalence class in $\mathscr{E}^*$ which contains A. An r.e. set A is *maximal* if A^* is a coatom (maximal element) of $\mathscr{E}^*$. Let Aut $\mathscr{E}$ (Aut $\mathscr{E}^*$) denote the group of automorphisms of $\mathscr{E}$ ($\mathscr{E}^*$). We prove that, for any two maximal sets A and B, there exists $\Phi \in \text{Aut}\,\mathscr{E}$ such that $\Phi(A)=B$. It follows that for each $k \geqq 1$ the group Aut $\mathscr{E}^*$ is k-ply transitive on its coatoms. This demonstrates much more uniformity of structure of $\mathscr{E}$ than was supposed, and answers a question of Martin and Lachlan [**1**, p. 36]. We also use automorphisms to relate the *structure* of an r.e. set to its *degree*, particularly for degrees $\boldsymbol{d}$ which are *high* ($\boldsymbol{d}'=\boldsymbol{0}''$) or *low* ($\boldsymbol{d}'=\boldsymbol{0}'$), and as corollaries we answer questions and extend results of Lachlan, Martin, Sacks, Yates, and others. The proofs involve infinite-injury priority arguments like those of Sacks [**11**], [**12**], and [**13**], but here an altogether different method is needed to resolve conflicts between opposing requirements. The numbering of results in §1 and §2 corresponds to that of [**15**] where full proofs will appear. The results in §3 will appear in [**16**] and [**17**].

1. **Background information.** For $A, B \in \mathscr{E}$, let $A \equiv_{\mathscr{E}} B$ ($A^* \equiv_{\mathscr{E}^*} B^*$) denote that there exists $\Phi \in \text{Aut}\,\mathscr{E}$ (Aut $\mathscr{E}^*$) such that $\Phi(A)=B$ ($\Phi(A^*)=B^*$). A permutation p of N *induces* an automorphism Φ of $\mathscr{E}$ ($\mathscr{E}^*$) if

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