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BIFURCATION AND STABILITY FOR A NONLINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATION

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This note is a brief report on some research conducted by the authors in 1971. A complete report on this same research is scheduled to appear in a separate article [**2**].

Let f be a given function continuously mapping the real line R into itself. Let λ be a given nonnegative real number. Let $\phi:[0,\pi]\rightarrow R$ be any C^1-smooth function such that $\phi(0)=\phi(\pi)=0$. We shall be discussing the following problem. Find a function u continuously mapping the domain $\{(x,t):0\leqq x\leqq\pi,\ 0\leqq t<+\infty\}$ into R such that (i) the partial derivative u_x is defined and continuous on $[0,\pi]\times[0,+\infty)$; (ii) the partial derivatives u_t and u_{xx} are defined and continuous on $[0,\pi]\times(0,+\infty)$; (iii) u satisfies the equations

$$\begin{array}{lll} \text{(1a)} & u_t(x,t)=u_{xx}(x,t)+\lambda f(u(x,t)) & (0\leqq x\leqq\pi,\ 0<t<+\infty) \\ \text{(1b)} & u(0,t)=u(\pi,t)=0 & (0\leqq t<+\infty) \\ \text{(1c)} & u(x,0)=\phi(x) & (0\leqq x\leqq\pi). \end{array}$$

By a solution of (1) we mean a function u having the properties just specified.

Our goal in studying (1) is to determine the asymptotic behavior of its solutions u as $t\rightarrow+\infty$. We shall discuss this asymptotic behavior in terms of bifurcation and stability phenomena exhibited by (1).

Other authors have studied this same type of problem for parabolic partial differential equations. Specifically, we mention the works by Keller and Cohen [7] and by Sattinger [**11**], [**12**]. The basic tool in these papers is the maximum principle for parabolic and elliptic partial differential equations. We also mention a recent paper by Auchmuty [**1**] in which the fundamental approach is the use of Liapunov methods. Our own work [**2**] to be described here is also based on Liapunov methods.

In our investigation we assume that f satisfies the following hypotheses:

(H_1) f is a C^2-smooth function mapping of R into itself.

(H_2) $f(0)=0$ and $f'(0)>0$.

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