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DYNAMICAL SYSTEMS, FILTRATIONS AND ENTROPY

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Introduction. I would like to try to pose some new directions in dynamical systems, at least in that part of the subject which deals with the qualitative behavior of the orbit structure of diffeomorphisms and flows of compact differentiable manifolds M without boundary. The birth of our first child, Alexander, to Beth and myself just two weeks before this talk has perhaps made me overly optimistic. But, I think that the subject can use some general approaches even if they be Pollyannic.

The basic problems that I will consider are:

(I) *Genericity*: That is, to find a generic set of diffeomorphisms or flows such that the asymptotic behavior of the orbits can somehow be reasonably understood.

A generic set means a subset which is a Baire set, that is, the countable intersection of open and dense sets.

(II) *Model making*: The idea here is to produce in some sense the best or simplest diffeomorphisms in each isotopy class of diffeomorphisms of M, that is, in each connected component of $\mathrm{Diff}^r(M)$, the group of C^r diffeomorphisms of M, $1 \leqq r \leqq \infty$. The properties that we shall say make a diffeomorphism f simplest are:

(a) f is structurally stable;

(b) f has the smallest topological entropy of any structurally stable diffeomorphism in its isotopy class.

Recall that $f \in \mathrm{Diff}^r(M)$ is structurally stable if there is a neighborhood of f, $U_f \subset \mathrm{Diff}^r(M)$, such that for any $g \in U_f$ there is a homeomorphism $h: M \to M$ with $hf = gh$. Structural stability says that up to continuous changes of variables the orbit structure of the diffeomorphisms in a neighborhood of f is locally constant.

Now to define entropy via a theorem of Bowen [**4**]. Let (X, d) be a compact metric space and $T: X \to X$ continuous. A set $E \subset X$ is (n, ε) separated if for any $x, y \in E$ with $x \neq y$ there is a j, $0 \leqq j < n$, such that $d(T^j(x), T^j(y)) > \varepsilon$. Let $S_n(\varepsilon)$ denote the largest cardinality of any (n, ε)

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