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GLOBAL DEFORMATION OF POLARIZED VARIETIES[1]

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1. Classically, the works of Riemann, Hurwitz, and Severi show that the set of complete nonsingular curves of genus g can be parametrized into an algebraic family of nonsingular curves, parametrized by an irreducible algebraic variety (allowing many curves of the family to be isomorphic to each other), and that isomorphism classes of these curves depend on $3g-3$ parameters ($g>1$). Moreover, two complete nonsingular curves of different genera cannot be members of an algebraic family parametrized by an irreducible algebraic variety. These imply, among other things, that the topological as well as the differential geometric nature of such curves can be characterized completely by the genus. Moreover, it is known from the result of Baily **[2]**, **[3]** that distinct isomorphism classes of such curves of genus g form an irreducible algebraic variety of dimension $3g-3$ (1 if $g=1$).

When we attempt to deal with similar problems for complete nonsingular and projectively embeddable varieties of higher dimensions, we encounter difficulties of higher magnitude. Limiting ourselves to the category of algebraic varieties for characteristic zero, Siegel **[18]**, Satake **[16]** and Baily **[3]** have shown that similar results are true for *polarized Abelian varieties*. We call a pair $(V, \mathscr{X})$ of a complete nonsingular variety V and a set $\mathscr{X}$ of V-divisors a polarized variety if $\mathscr{X}$ satisfies the following conditions:

(i) $\mathscr{X}$ contains a nondegenerate divisor X (ample in the sense of **[5]**);

(ii) A V-divisor Y is in $\mathscr{X}$ if and only if $rY\equiv sX$ (numerical equivalence) for some integers r, s.

A divisor Y in $\mathscr{X}$ is called a *polar divisor*. $\mathscr{X}$ contains a divisor X_0 such that a V-divisor Y is in $\mathscr{X}$ if and only if $Y\equiv sX_0$ for some integer s and a nondegenerate polar divisor X can be expressed as $X\equiv rX_0$ with a positive integer r (cf. **[10]**). Such a polar divisor X_0 is called a *basic polar divisor*. Already there are many examples of algebraic surfaces which indicate

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