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THE SINGULARITIES OF THE $\mathcal{S}$-MATRIX AND GREEN'S FUNCTION ASSOCIATED WITH PERTURBATIONS OF $-\Delta$ ACTING IN A CYLINDER

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It is the purpose of this note to study the singularities of the $\mathcal{S}$-matrix and Green's function associated with the operators considered in [1]–[3]. As will be seen, there are a countable number of branch points, as well as a countable number of different $\mathcal{S}$-matrices associated with these operators. In this respect, these results differ considerably from those drawn from quantum mechanical scattering[2] and the exterior problem (see e.g. [4] and [5]).

1. **Preliminaries.** Let S denote the semi-infinite cylinder in R^N, N-dimensional Euclidean space ($N \geqq 2$), with arbitrary bounded, smooth $N-1$ dimensional cross-section l. Thus S consists of the points $x = ((x_1, \ldots, x_{N-1}), x_N) = (\tilde{x}, x_N)$, where $\tilde{x} \in l$ and $x_N \geqq 0$.[3] Let Ω denote the domain with smooth boundary $\dot{\Omega}$, obtained from S by perturbing a finite part of $\dot{S}$. Thus $\Omega = S$ for $x_N \geqq \mathring{x}_N$ for some fixed $\mathring{x}_N > 0$.

We now define the operators $A_0(A)$ by $-\Delta$ acting in $L_2(S)$ $(L_2(\Omega))$ and associated with zero Dirichlet boundary conditions on $\dot{S}(\dot{\Omega})$. Let A_l denote the corresponding operator defined in $L_2(l)$ and let $\{\nu_n\}$ and $\eta_n(\tilde{x})$ denote a complete set of eigenvalues (in increasing order) and corresponding orthonormal eigenfunctions for A_l. Let A^c denote that part of A orthogonal to all of its eigenvalues, Λ denote the set of eigenvalues of A and $\Lambda' = \Lambda \cup \{\nu_n\}$.

It was shown in [1] that a complete set of generalized eigenfunctions for A_0 and A^c are given by

$$w_n^0(x; \lambda) = (2/\pi)^{1/2} \sin(\lambda - \nu_n)^{1/2} x_N \eta_n(\tilde{x}), \qquad \lambda \notin \{\nu_n\},$$

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[1] Work performed under the joint auspices of the Atomic Energy Commission and the Mathematics Research Center at the University of Wisconsin.

[2] Our results are related to wave progagation in a waveguide.

[3] We might just as easily consider the infinite cylinder, $S' = (x = (\tilde{x}, x_N) \mid \tilde{x} \in l, -\infty < x_N < \infty)$.