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A COHOMOLOGY FOR FOLIATED MANIFOLDS

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1. **Introduction.** Let M be a connected manifold and τ a foliation on M. τ is then an involutive subbundle of TM, the tangent bundle of M. Denote by ν the normal bundle to τ, $\nu = TM/\tau$. We denote sections of a bundle P over M by $\Gamma(P)$. All manifolds, bundles and maps are assumed to be C^∞.

There is a canonical connection ∇ on ν which is flat along τ [**B**]. Consider the complex

$$\Gamma(\nu) \xrightarrow{\hat{d}} \Gamma(\nu \otimes \Lambda^1\tau^*) \xrightarrow{\hat{d}} \Gamma(\nu \otimes \Lambda^2\tau^*) \xrightarrow{\hat{d}} \cdots,$$

where τ^* is the cotangent bundle to the foliation and

$$\begin{aligned}\hat{d}(\sigma)(X_1, \ldots, X_{k+1}) &= \sum_{1 \leq i \leq k+1} (-1)^i \nabla_{X_i}(\sigma(X_1, \ldots, \hat{X}_i, \ldots, X_{k+1})) \\ &+ \sum_{1 \leq i < j \leq k+1} (-1)^{i+j+1}\sigma([X_i, X_j], X_1, \ldots, \hat{X}_i, \ldots, \hat{X}_j, \ldots, X_{k+1})\end{aligned}$$

for $\sigma \in \Gamma(\nu \otimes \Lambda^k\tau^*)$, $X_1, \ldots, X_{k+1} \in \Gamma(\tau)$.

Since the curvature tensor of ∇ restricted to τ is identically zero we have that $\hat{d} \circ \hat{d} = 0$. Denote the homology of this complex by $F^*(\tau; \nu)$. This is the cohomology of the Lie algebra of vector fields tangent to the foliation with coefficients in sections of the normal bundle, the representation being given by the connection [**GF**].

In general the groups $F^k(\tau; \nu)$ are not finitely generated (the complex is not elliptic) but they satisfy the following.

(i) F^* is a functor from the category of foliated manifolds and transverse maps to the category of abelian groups and homomorphisms.

(ii) If $f: N \to M$ is an embedded transverse submanifold, we can define relative cohomology groups $F^*(\tau; \nu, f)$ and obtain the usual long exact sequence.

(iii) F^* is an invariant of the diffeomorphism type of the foliation. However, F^* is not an invariant of the integrable homotopy type of the foliation when M is an open manifold.

2. **Interpretation of $F^1(\tau; \nu)$.** Fix a Riemannian metric on M and think

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