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# DECOMPOSITIONS OF MODULES AND MATRICES

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ABSTRACT A canonical form for a module $M$ over a commutative ring $R$ is a decomposition $M \cong R/I_1 \oplus \cdots \oplus R/I_n$, where the $I_j$ are ideals of $R$ and $I_1 \subseteqq \cdots \subseteqq I_n$. A complete structure theory is developed for those rings for which every finitely generated module has a canonical form. The (possibly larger) class of rings, for which every finitely generated module is a direct sum of cyclics, is also considered, and partial results are obtained for rings with fewer than $2^c$ prime ideals. For example, if $R$ is countable and every finitely generated $R$-module is a direct sum of cyclics, then $R$ is a principal ideal ring. Finally, some topological criteria are given for Hermite rings and elementary divisor rings.

All rings in this announcement are commutative with 1, and all modules are unital. A *canonical form* for an $R$-module $M$ is a decomposition $M \cong R/I_1 \oplus \cdots \oplus R/I_n$, where $I_1 \subseteqq \cdots \subseteqq I_n \neq R$. If $M$ has a canonical form, the ideals $I_j$ are uniquely determined [**K**]. A CF-ring is a ring for which every finitely generated direct sum of cyclics has a canonical form. It can be shown that $R$ is CF if and only if

$$R/I \oplus R/J \cong R/(I \cap J) \oplus R/(I + J)$$

for every pair of ideals $I$, $J$.

By a valuation ring we shall mean a ring, possibly with zero-divisors, whose lattice of ideals is totally ordered. A ring $R$ is arithmetical, provided the local ring $R_{\mathfrak{m}}$ is a valuation ring for each maximal ideal $\mathfrak{m}$. Finally, an $h$-local domain [**M1**] is an integral domain such that (1) every nonzero ideal is contained in only finitely many maximal ideals, and (2) every nonzero prime ideal is contained in a unique maximal ideal.

THEOREM 1. *Every CF-ring is a finite direct product of indecomposable CF-rings. The indecomposable CF-rings are precisely the rings $R$ such that* (i) *$R$ is arithmetical,* (ii) *$R$ has a unique minimal prime $P$,* (iii) *$R/P$ is an $h$-local domain, and* (iv) *every ideal contained in $P$ is comparable with every ideal of $R$.*

Thus valuation rings and arithmetical $h$-local domains are CF-rings.

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