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OBSTRUCTIONS IN SHAPE THEORY

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1. **Introduction.** The classical theory of obstructions (see, for instance, Spanier [**5**]) applied only to the case where all the spaces concerned have the homotopy type of a CW-complex, however the work on shape theory by the group of Polish topologists around Borsuk and, more recently, by Mardešić and Segal (see [**2**]) has indicated that it is possible to adapt classical homotopy theory to give useful information on, for instance, general compact Hausdorff spaces. At the same time, the author has, independently, developed a rudimentary homotopy theory [**3**] which complements the classical Čech homology and cohomology theories. This theory has close connections with Borsuk's theory, but has more of the flavour of Artin and Mazur's work on the étale homotopy theory in algebraic geometry. (The connections with shape theory are explored in [**4**].)

The attack of both these approaches has been restricted, albeit successfully, to the relatively simple parts of homotopy theory; it therefore seemed natural to attack one of the deeper areas from a similar viewpoint as a next step in the development of the theories. This note announces the initial results of my attempt to study the extension and lifting problems from this point of view. The details will, I hope, be published later.

2. **A cohomology theory on a category of diagrams.** Although the methods of both shape theory and Čech homotopy use inverse systems of polyhedra, it is simpler to develop our obstruction theory in a category of diagrams of pointed CW-complexes and to translate to the more general case later.

The category of pointed CW-complexes and cellular maps is denoted by CW_* and Σ will denote the ordered linear category of positive integers where there is a unique map from m to n as soon as $m \geqq n$.

Suppose $F:\Sigma \to Ab$ is a diagram of abelian groups and $C_*:\Sigma \to C(Ab)$ a diagram of chain complexes of abelian groups. A collection

$$f = \{f(\sigma):C_n(\sigma) \to F(\sigma)\}_\sigma$$

will be called a weak n-cochain if for each morphism $\alpha:\sigma \to \tau$ in Σ, there

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