THE ALGEBRA OF BOUNDED ANALYTIC FUNCTIONS

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Introduction. This survey features the algebra $H^\infty(D)$ of bounded analytic functions on a domain D. The central theme will be the interplay between classical function theory and functional analysis.

Typically one learns about the spectrum of $H^\infty(D)$, or some other abstract entity, by rephrasing the problem in terms of function theory and treating the reformulated problem with techniques of classical hard analysis. Problems of this sort have led to difficult and fecund work in hard analysis, the most notable example being Carleson's solution of the corona problem [6].

In the reverse direction, uniform-algebra techniques are beginning to yield modest but new results in function theory. An example [18] is the strong version of the Iversen-Tsuji theorem, valid for polydomains, which can be obtained from a statement concerning the Šilov boundary of $H^\infty(D)$. Moreover, the abstract concepts, such as "distinguished homomorphism," have served to enhance our understanding of the classical function theory.

The Banach-algebra approach to $H^\infty(D)$ really got under way during the Conference in Analytic Function Theory at Princeton University in 1957. At that time, virtually nothing was known about the maximal ideal space $\mathcal{M}(D)$ of $H^\infty(D)$. The problems brought to this conference included the following questions. Do there exist interpolating sequences in the open unit disc Δ for $H^\infty(\Delta)$, that is, are there sequences S in Δ such that $H^\infty(\Delta)|_S = l^\infty(S)$? Does $\mathcal{M}(\Delta)\backslash\Delta$ coincide with the Šilov boundary of $H^\infty(\Delta)$? Is Δ dense in $\mathcal{M}(\Delta)$? This latter problem was christened the "corona problem," the "corona" being the part of $\mathcal{M}(\Delta)$ which is not adherent to Δ.

This area has seen extensive development since the 1957 conference, and our aim is to give an indication of where we stand in year 15. However, this is not to be a complete survey, and the selection of topics and of bibliographical references will be seen to be quite prejudiced. Many

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