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THE SPECTRA FOR OPERATORS OF A BASIC COLLECTION

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We present here the spectra of operators from a basic collection considered in the scale of Lebesgue spaces of pth power summable functions over a finite interval.

Without loss of generality we confine our attention to complex valued functions over the interval $[0, 1]$. The associated Banach spaces are denoted by L^p, $1 < p < \infty$. The results lift to any underlying bounded interval $[a, b]$ through, for example, the mappings induced by the linear map $[a, b] \ni s \mapsto (b - a)^{-1}(s - a) = t \in [0, 1]$.

The results unfold primarily through certain formal manipulations on some basic relations in an algebra of elementary operations.

On complex valued functions over the interval $[0, 1]$ we consider the operations (see [**2**])

$$\begin{aligned} J^{\beta}\psi(t) &= \Gamma(\beta)^{-1} \int_0^t (t - x)^{\beta - 1}\psi(x)\,dx, && 0 < \operatorname{Re}\beta, \\ &= \lim_{b \to 0+} J^{b+\beta}\psi(t) \quad (L^p\text{-limit}), && \operatorname{Re}\beta = 0, \\ &= dJ^{\beta+1}\psi(t)/dt, && -1 < \operatorname{Re}\beta < 0, \end{aligned}$$

and

$$\begin{aligned} J^{*\beta}\psi(t) &= \Gamma(\beta)^{-1} \int_t^1 (x - t)^{\beta - 1}\psi(x)\,dx, && 0 < \operatorname{Re}\beta, \\ &= \lim_{b \to 0+} J^{*b+\beta}\psi(t) \quad (L^p\text{-limit}), && \operatorname{Re}\beta = 0, \\ &= -dJ^{*\beta+1}\psi(t)/dt, && -1 < \operatorname{Re}\beta < 0. \end{aligned}$$

In addition let M^{γ} denote the operation given by $M^{\gamma}\psi(t) = t^{\gamma}\psi(t)$, γ-complex, and R the operation $R\psi(t) = \psi(1 - t)$. Further denote by H the finite Hilbert transform

$$H\psi(t) = \frac{1}{\pi}(\text{p.v.}) \int_0^1 \frac{\psi(x)}{t - x}\,dx,$$

the integral being the Cauchy principal value. We consider also H as

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