BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 4, July 1973

BASIS GRAPHS OF PREGEOMETRIES

BY STEPHEN B. MAURER

Communicated by Gian-Carlo Rota, December 18, 1972

A **combinatorial pregeometry**, or **matroid**, may be defined as a finite set of **elements** E and a collection of **bases** $\mathscr{B}$, all subsets of E, such that for all $B,B' \in \mathscr{B}$ and any $e' \in B' - B$, there exists $e \in B - B'$ for which $B - e + e' \in \mathscr{B}$. This exchange axiom suggests it is fruitful to represent a pregeometry $\mathscr{M}$ by a graph: Let there be a vertex for each basis and an edge for each pair of bases differing by a single exchange. We get the **basis graph** $BG(\mathscr{M})$. A special case of this construct, tree graphs, has been studied for several years [**3**]. The more general situation has attracted attention only recently [**1**], [**4**].

Our purpose here is to announce our own studies of pregeometry basis graphs [**6**], [**7**], and to state some of our key findings. We have recently learned that some of our results and methods are similar to those discovered about the same time by Cunningham [**2**] and also by Holzmann, Norton and Tobey [**5**]. In particular, Theorems 2 and 3 below are in this category.

We first characterize basis graphs. Given any graph $G(\mathscr{V}, \mathscr{E})$, suppose $\delta(v', v'') = 2$ and $\mathscr{V}' \subset \mathscr{V}$ consists of v', v'' and all vertices adjacent to both. Then the induced subgraph $\langle \mathscr{V}' \rangle$ is called the **common neighbor subgraph** $CN(v', v'')$, or simply a CN. In a basis graph each CN is either a square (4-cycle), a pyramid (with square base), or an octahedron. Again in any graph, a **leveling** from v_0 is a partition of $\mathscr{V}$ into

$$\mathscr{V}_k = \{v : \delta(v, v_0) = k\}, \qquad k = 0, 1, \ldots.$$

In any leveling of a basis graph, each octahedral CN lies in one of three positions: (i) all in one level; (ii) across two levels, three adjacent vertices in each; or (iii) across three levels, one vertex in the highest, one not adjacent to it in the lowest, and four in between. Any other CN must lie as would an induced subgraph of such an octahedron. We call this the **positioning condition**. Finally, the **neighborhood subgraph** $N(v)$ is the induced subgraph on the vertices adjacent to v (v not included).

THEOREM 1. *G is a basis graph iff*

(1) *it is connected*;

(2) *each CN is a square, pyramid or octahedron*;

AMS (MOS) subject classifications (1970). Primary 05B35; Secondary 05C05, 05C35, 05B20, 15A30.

Key words and phrases. Pregeometry, matroid, basis graph, tree graph, combivalence class, pivoting.