BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 4, July 1973

THE PROBABILITY OF CONNECTEDNESS OF A LARGE UNLABELLED GRAPH[1]

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Communicated by Gian-Carlo Rota, December 18, 1972

An (n, q) graph is one with n nodes and q edges, in which any two different nodes are or are not joined by a single edge. We write $T = T(n, q)$ for the number of different (n, q) graphs with unlabelled nodes and t for the number of these graphs which are connected, so that $\beta = t/T$ is the probability that an unlabelled (n, q) graph is connected. We write F, f and α for the corresponding numbers for (n, q) graphs whose nodes are labelled. We write also $N = n(n-1)/2$, $B(h, k) = h!/\{k!(h-k)!\}$ and $\gamma = (2q - n\log n)/n$. Clearly $q \leqq N$. In what follows, A (not always the same at each occurrence) is a fixed positive number at our choice and all statements are true only for $n > n_0, q > q_0$, where n_0 and q_0 depend on the A.

Erdös and Renyi [**1**] put $q = [n(\log n + a)/2]$, where a is independent of n and q, and showed that, for these q, we have

$$\alpha \to \exp(e^{-a}) \tag{1}$$

as $n \to \infty$. For given n, it can be shown trivially that α increases steadily (in the nonstrict sense) as q increases. Hence, from (1), it can be at once deduced that, as $n \to \infty$, we have $\alpha \sim \exp(e^{-\gamma})$ and, in particular, that

$$\alpha \to 1 \quad (\gamma \to +\infty), \qquad \alpha \to 0 \quad (\gamma \to -\infty).$$

Elsewhere [**4**] I have shown that, if $\gamma \to +\infty$, then f has an asymptotic expansion of which the first two terms are

$$f = B(N, q) - nB(N - n + 1, q) - \cdots.$$

Now $F = B(N, q)$ and

$$\frac{nB(N-n+1, q)}{B(N, q)} = n\prod_{s=0}^{q-1} \frac{N-n+1-s}{N-s} \leqq n(N-n+1)^q N^{-q}$$

and the logarithm of this is less than $\log n - \{q(n-1)/N\} = -\gamma$. Hence my result leads to $\alpha = 1 - O(e^{-\gamma})$, a statement which is only nontrivial

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AMS (MOS) subject classifications (1970). Primary 05C30.

Key words and phrases. Unlabelled graphs, asymptotic enumeration, connectedness, probability of connectedness.

[1] The research reported herein was sponsored in part by the United States Government.