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# UNIQUENESS OF ORIENTATION PRESERVING PL INVOLUTIONS OF 3-SPACE

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1. **Introduction.** Waldhausen [**2**] has proven that every PL involution of $S^3$ with 1-dimensional fixed point set is PL equivalent to the one which rotates $S^3$ around an unknotted simple closed curve. In this note we show how the corresponding result for $R^3$ (which has been heretofore unknown) follows from a technique used by the second author in his recent paper [**1**]. Specifically, we prove

THEOREM 1. *Every orientation preserving PL involution of $R^3$ is PL equivalent to the one which rotates $R^3$ around the z-axis.*

Since such an involution must have 1-dimensional fixed point set, the above theorem is a consequence of the following theorem if one considers the one-point compactification of $R^3$.

THEOREM 2. *Let $h$ be an involution of a closed 3-manifold $M$ with 1-dimensional fixed point set $F$. If, for some $x \in F$, there exists a triangulation of $M - \{x\}$ making $h|M - \{x\}$ piecewise linear, then there exists a triangulation of $M$ making $h$ piecewise linear.*

Theorem 2 will be proved by literally imitating the reduction method [**2**, proof of Lemma 2] of Tollefson.

2. **Proof of Theorem 2.**

LEMMA. *Let $M$, $F$, $x$ and $h$ be as in Theorem 2. Then, for any neighborhood $U$ of $x$, there exists in $U$ an invariant 3-cell $D$ containing $x$ in its interior such that $\partial D \cap F \neq \emptyset$ and $\partial D$ is a PL subspace of $M - \{x\}$.*

PROOF. We indicate how to modify the proof of Lemma 2 of [**1**] to produce the desired invariant 3-cell $D$. We may assume that $F$ is not contained in $U$. Let $\Sigma$ be the set of all PL 2-spheres in $M - \{x\}$ that bound 3-cell neighborhoods of $x$ in $U$ and are in $h$-general position modulo $F$ (in the sense of [**1**]). The lemma follows from the proof of Lemma 2 of [**1**] if the phrase "2-spheres not bounding 3-cells" is replaced by "PL 2-spheres in $M - \{x\}$ bounding 3-cell neighborhoods of $x$ in $U$."

In order to prove Theorem 2, consider a sequence of invariant 3-cells

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