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THE VITALI-HAHN-SAKS AND NIKODYM THEOREMS FOR ADDITIVE FUNCTIONS. II

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ABSTRACT. In this note appropriate versions of the Vitali-Hahn-Saks and Nikodym theorems are established for s-bounded additive set functions with values in an abelian topological group G.

Although we shall use $+$ and 0 to denote addition and identity in both G and R, the real numbers, no confusion should arise. Thus we denote by $\mathscr{U}$ the set of symmetric neighborhoods of 0 in G, and for $U \in \mathscr{U}$ we set $1U = U$ and $(n+1)U = \{x + y : x \in nU, y \in U\}$, $n \in N$, the set of positive integers.

A subset H of G is said to be bounded if for each $U \in \mathscr{U}$ there exists $n \in N$ such that $H \subset nU$.

We suppose that finite subsets of G are bounded.

Let Ω be a nonempty set and let $\mathscr{S}$ be a sigma algebra of subsets of Ω.

A function μ from $\mathscr{S}$ to G is said to be additive if $\mu(\phi) = 0$ and $\mu(E \cup F) + \mu(E \cap F) = \mu(E) + \mu(F)$, $E, F \in \mathscr{S}$.

An additive function μ is said to be s-bounded (cf. [**1**],[**5**]) if $\lim_n \mu(E_n) = 0$ (i.e., for each $U \in \mathscr{U}$ there is $m \in N$ such that $\mu(E_n) \in U, n > m$) for each sequence $\{E_n\}$ of pairwise disjoint elements of $\mathscr{S}$.

Notice that if μ is s-bounded, $U \in \mathscr{U}$, and $\{E_n\}$ is a sequence of pairwise disjoint elements of $\mathscr{S}$, then there exists $n \in N$ such that if M is a finite subset of $N^n = \{k \in N : k \geqq n\}$ then $(\sum_{i \in M} \mu(E_i)) \in U$.

An additive function μ is said to be bounded if $\mu(\mathscr{S}) = \{\mu(E) : E \in \mathscr{S}\}$ is a bounded subset of G.

For the case when μ is sigma-additive, versions of our results can be found in [**4**]; for the case where μ is merely additive and $G = R$, one can refer to [**2**].

Our version of Nikodym's theorem, a striking improvement of the principle of uniform boundedness, follows.

THEOREM 1. *Suppose that T is a set of s-bounded functions such that for each $E \in \mathscr{S}$ the set $T(E) = \{\mu(E) : \mu \in T\}$ is bounded, then $T(\mathscr{S}) = \{\mu(E) : \mu \in T, E \in \mathscr{S}\}$ is bounded.*

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