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A NOTE ON WITT RINGS

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This note contains some applications of the theory of Mackey functors (cf. [3], [4] and [5]) to the study of Witt rings. A detailed version may be found in [3, Appendices A and B].

So let R be a commutative ring with $1 \in R$ and $W(R)$ its Witt ring as defined in [7]. Any ring homomorphism $\rho: R \to R'$ defines a ring homomorphism $\rho_*: W(R) \to W(R')$. Moreover if R' is separable over R and finitely generated projective as an R-module (let ρ be called admissible in this case), the trace map $R' \to R$ defines a $W(R)$-linear map backwards: $\rho^*: W(R') \to W(R)$ (cf. [1] and [12], [13]). These observations lead easily to

PROPOSITION 1. *Let $\mathfrak{C}$ be the category with objects the commutative rings R (with $1 \in R$) and with morphisms $[R', R]_{\mathfrak{C}} = \{\rho: R \to R' \mid \rho \text{ admissible}\}$ (i.e. $\mathfrak{C}$ is dual to the category of commutative rings with admissible maps). Then the Witt ring construction defines a Mackey functor $W: \mathfrak{C} \to \mathcal{Ab}$, the category of abelian groups, together with a commutative, associative and unitary inner composition, given by the multiplication in the Witt ring.*

COROLLARY 1. *Let $\rho: R \to R'$ be admissible and $n \cdot 1_{W(R)} \in \rho^*(W(R'))$ for some $n \in N$. Then all the "Amitsur cohomology groups" $H^i(R'/R, W)$ (i.e. the cohomology groups of the semisimplicial complex $0 \to W(R) \to W(R') \rightrightarrows W(R' \otimes_R R') \Rrightarrow W(R' \otimes_R R' \otimes_R R') \cdots$) are n-torsion groups, especially for $n = 1$ they are all trivial.*

PROOF. Apply the results of [5] to this special situation (they were found precisely to be applied right here!).

Examples of admissible maps $\rho: R \to R'$ with $1_{W(R)} \in \rho^*(W(R'))$ have been given by Scharlau (cf. [12] and [3, Appendix A, Lemmas 2.3, 2.4, 2.5]).

As a rather special case we get this way:

COROLLARY 2 (CF. [11] AND [8]). *Let L/K be a finite Galois extension (of fields) of odd degree and with Galois group G. Then the natural action of G on $W(L)$ has trivial (co)homology:*

$$H^0(G, W(L)) \cong H_0(G, W(L)) \cong W(K),$$

$$H^i(G, W(L)) = H_i(G, W(L)) = \hat{H}^j(G, W(L)) = 0 \qquad (i \geqslant 1, j \in Z).$$

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