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THE CHARACTERS OF THE BINARY MODULAR CONGRUENCE GROUP

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1. **Introduction.** The determination of the characters of the groups $SL(2, Z/p^nZ)$ where p is an odd prime is of interest for several reasons, among them the role that the group plays in the study of elliptic modular functions [**1**], [**4**], [**5**], [**6**]. Attempts have been made to compute the values of the irreducible characters of $SL(2, Z/p^nZ)$ and complete results were obtained by Shur [**11**] in the case $n = 1$ and Praetorius [**9**] and Rohrbach [**10**] in the case $n = 2$. Since that time analytic techniques involving theta-functions [**7**] and methods used for similar problems over locally compact groups [**12**] have been applied with partial results in the first case and a classification theorem in the second.

The purpose of the note is to announce that a complete description of the irreducible representations of $SL(2, Z/p^nZ)$ as well as the computation of the characters of these representations has been obtained by the author. These results comprise the author's Ph.D. Thesis (University of Wisconsin—1972) and will be published elsewhere.

2. **Outline of results.** Write G_n for L.F.$(2, Z/p^nZ)$; i.e., let

$$G_n = SL(2, Z/p^nZ)/\{\pm\left(\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}\right)\}.$$

For $r = 1, 2, \ldots, n$, let $K_{n,r}$ be the kernel of the homomorphism of G_n onto G_r that is the result of "reading" G_n modulo p^r. Then $K_{n,r}$ is abelian of type $(p^{n-r}, p^{n-r}, p^{n-r})$ whenever $r \geqq n/2$. Now fix n to be even and let $r = n/2$. Write $e_r(x)$ for $\exp(2\pi ix/p^r)$.

DEFINITION 1. Fix l, $0 \leqq l \leqq p^r - 1$ and write $l = p^\alpha l_1$ where $0 \leqq l_1 \leqq p^{r-\alpha} - 1$ and $p \nmid l_1$ in case $l \neq 0$ and where we take $\alpha = r$, $l_1 = 1$ in case $l = 0$. Write a typical element of $K_{n,r}$ in the form $I + p^r\left(\begin{smallmatrix}a & b\\ c & -a\end{smallmatrix}\right)$ where $I = \left(\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}\right)$. Then ψ_l is defined to be the character on $K_{n,r}$ which maps $I + p^r\left(\begin{smallmatrix}a & b\\ c & -a\end{smallmatrix}\right)$ to $e_r((lb + c)/2l_1)$.

THEOREM 1. *ψ_l may be extended to a character η_l defined on the normalizer of ψ_l (see* [**4**] *for definitions). The character χ_l induced by η_l on G_n is irreducible. χ_l does not contain $K_{n,n-1}$ in its kernel.*

Having proved Theorem 1 it is a simple matter to determine con-

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