BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 3, May 1973

CLASS NUMBERS OF DEFINITE QUATERNARY FORMS WITH NONSQUARE DISCRIMINANT

BY PAUL PONOMAREV[1]

Communicated by Nathan Jacobson, November 2, 1972

1. **Introduction.** Let V be a definite quadratic space of dimension four over the field of rational numbers $\boldsymbol{Q}$. If the discriminant $\Delta(V)$ is square, then the number of classes of maximal integral lattices can be computed by means of the formulas given by the author in [**6**] and [**7**]. The purpose of this note is to announce analogous class number formulas for the case where $\Delta(V)$ is not square, V_p is isotropic for each finite prime p, and the norm of the fundamental unit of $K = \boldsymbol{Q}((\Delta(V))^{1/2})$ is -1.

Let $\mathfrak{M}$ denote the genus of maximal integral lattices of V, $\mathfrak{J}$ the idealcomplex containing $\mathfrak{M}$. Let Δ denote the discriminant of $\mathfrak{M}$. Then $\mathfrak{J}$ can also be described as the set of all maximal lattices having reduced determinant Δ [**1**, p. 87]. The formulas we present here are for the number of (proper) similitude classes in $\mathfrak{J}$. We denote this class number by H. The number of classes in $\mathfrak{M}$ will be denoted by H_0. The idealcomplex $\mathfrak{J}$ decomposes into g^+ similitude genera, where $g^+ =$ the number of strict genera of K [**5**, p. 338]. The similitude genus containing $\mathfrak{M}$ has H_0 similitude classes. It follows that $H_0 \leqq H$. Equality holds if and only if K has prime discriminant, since $g^+ = 1$ in that case (cf. Corollary below).

2. **Preliminaries.** Let C_V^+ denote the second Clifford algebra of V. Then $C_V^+ = \mathfrak{A}_K = \mathfrak{A} \otimes_{\boldsymbol{Q}} K$, where $\mathfrak{A}$ is a definite quaternion algebra over $\boldsymbol{Q}$. Let $\alpha \mapsto \alpha^*$ be the canonical involution of $\mathfrak{A}_K$ and $N : \mathfrak{A}_K \to K$ the (reduced) norm mapping. The conjugation $x \mapsto \bar{x}$ of K can be extended to a $\boldsymbol{Q}$-automorphism $\alpha \mapsto \bar{\alpha}$ of $\mathfrak{A}_K$ so that $\mathfrak{A}$ is its ring of fixed elements. Let W be the set of all α in $\mathfrak{A}_K$ such that $\alpha = \bar{\alpha}^*$. Then W is a four-dimensional $\boldsymbol{Q}$-subspace of $\mathfrak{A}_K$ and the restriction of N to W takes values in $\boldsymbol{Q}$. In this way W may be regarded as a quadratic space over $\boldsymbol{Q}$. We assume, without loss of generality, that V is positive definite. Then V is isometric to W. In particular, the condition that V_p is isotropic for every finite prime p is equivalent to the condition that $\mathfrak{A}$ splits at every finite prime which splits in K. From this it follows, by a straightforward computation of local discriminants, that

$$\Delta = \Delta_K(p_1 \cdots p_e)^2, \tag{1}$$

where Δ_K is the discriminant of K and $p_1, \ldots, p_e$ are all the nonsplit finite

AMS (MOS) subject classifications (1970). Primary 10C05, 12A80; Secondary 12A85.
[1] Partially supported by NSF grant GP-25320.