

chains. An *addition chain for n* is a sequence

$$1 = a_0, a_1, \dots, a_r = n$$

such that

$$a_i = a_j + a_k \quad \text{for some } k \leq j < i.$$

Most work on addition chains is concerned with the calculation of $l(n)$, the length of the shortest addition chain for n . The relationship of these chains to computer algorithms is that an $l(n)$ -chain gives a way to form x^n , arbitrary x and integral n , with fewest multiplications. Of special interest to mathematicians is the still unanswered "Scholz-Brauer conjecture"

$$l(2^n - 1) \leq n - 1 + l(n).$$

Knuth's paper consolidates relevant mathematical and empirical work on addition chains; it is certainly of more interest to the mathematician than to the computer scientist. Recent computer work by Knuth (appearing in the second printing) has turned up the remarkable fact that $l(12509) < l^*(12509)$ —an l^* -chain is one in which $j = i - 1$.

4. For various reasons, many of them nontechnical and extraneous, the interaction between mathematics and computer science over the past two decades has been less than one might expect. This has been and is unfortunate as each field can significantly benefit from and contribute to the other. The real importance of Knuth's work is that it represents a truly positive step towards eliminating the existing breach between mathematicians and computer scientists.

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