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BOUNDED HARMONIC BUT NO DIRICHLET-FINITE HARMONIC

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ABSTRACT. The purpose of the present note is to announce that for each $n \geqq 3$ there exists a Riemannian n-manifold, which carries nonconstant bounded harmonic functions but no nonconstant Dirichlet-finite harmonic functions.

1. A C^2-function u on a Riemannian n-manifold M is harmonic on M if $\Delta u = 0$, where

$$\Delta u = \frac{-1}{g^{1/2}} \sum_{i,j=1}^{n} \frac{\partial}{\partial x^i}\left(g^{1/2} g^{ij} \frac{\partial u}{\partial x^j} \right).$$

Here (g_{ij}) is the metric tensor for M, $(g^{ij}) = (g_{ij})^{-1}$, and $g = \det(g_{ij})$.

It is not known (cf. Sario-Nakai [**4**, p. 406]) whether or not for each $n \geqq 3$ there exists a Riemannian n-manifold M which carries nonconstant bounded harmonic functions but every harmonic function u on M is a constant whenever its Dirichlet integral

$$D(u) = \int_M \sum_{i,j=1}^{n} g^{ij} \frac{\partial u}{\partial x^i} \frac{\partial u}{\partial x^j} \, dV < \infty,$$

where $dV = g^{1/2}\, dx^1 \wedge dx^2 \wedge \cdots \wedge dx^n$ is the volume element. For $n = 2$ the problem was solved in the affirmative by Tôki [**5**], his example known as Tôki's example. Royden [**2**] and Sario [**3**] also obtained a similar result.

The purpose of the present note is to announce that for each $n \geqq 3$ there does exist a Riemannian n-manifold which solves the problem in the affirmative.[1] Details will be published elsewhere.

2. Fix $n \geqq 3$. Denote by M_0 the punctured Euclidean n-space $R^n - 0$ with the metric tensor

$$g_{ij}(x) = |x|^{-4}(1 + |x|^{n-2})^{4/(n-2)}\delta_{ij}, \quad 1 \leqq i,j \leqq n,$$

where $|x| = [\sum_{i=1}^n (x^i)^2]^{1/2}$ for $x = (x^1, x^2, \ldots, x^n) \in M_0$.

LEMMA. *Every positive harmonic function u on M_0 has the form*:

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[1] Professor Sario informed me that he recently obtained a similar result with Professors Wang and Hada.