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STOCHASTIC INTEGRALS AND PARABOLIC EQUATIONS IN ABSTRACT WIENER SPACE

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Kuo [2] has developed a theory of stochastic integrals and Piech [3] has established the existence of fundamental solutions of a class of parabolic equations, both working within the context of abstract Wiener space. In this note we establish the relationship between the work of Kuo and Piech, and as a consequence of this relationship we obtain a uniqueness theorem for fundamental solutions. We also provide a new proof of the non-negativity and semigroup properties of fundamental solutions.

Let H be a real separable Hilbert space, with inner product $(\ ,\ )$ and norm $|\cdot|$; let $\|\cdot\|$ be a fixed measurable norm on H; let B be the completion of H with respect to $\|\cdot\|$; and let i denote the natural injection of H into B. The triple (H, B, i) is an abstract Wiener space in the sense of Gross [1]. We may regard $B^* \subset H^* \approx H \subset B$ in the natural fashion. A bounded linear operator from B to B^* may thus be viewed as an operator on B or, by restriction to H, as an operator on H. The restriction to H of a member T of $L(B, B^*)$ is of trace class in $L(H)$ $(\equiv L(H, H))$ and

$$\|T_{|H}\|_{\mathrm{Tr}} \leqq \text{constant} \cdot \|T\|_{L(B,B^*)}.$$

Where no confusion of interpretation is possible, we will use T for $T_{|H}$. In order to work with stochastic integrals on (H, B, i) we formulate the following hypothesis:

(h) There exists an increasing sequence $\{P_n\}$ of finite dimensional projections on B such that $P_n[B] \subset B^*$, $\{P_n\}$ converges strongly to the identity on B, and $\{P_{n|H}\}$ converges strongly to the identity on H.

For $t > 0$, let p_t denote the Wiener measure on the Borel field of B which is determined by Gauss cylinder set measure on H of variance parameter t. Let Ω be the space of continuous functions ω from $[0, \infty)$ into B and vanishing at zero, and let $\mathscr{M}$ be the σ-field of Ω generated by the functions $\omega \to \omega(t)$. Then there is a unique probability measure $\mathscr{P}$ on $\mathscr{M}$ for which the condition $0 = t_0 < t_1 < \cdots < t_n$ implies that $\omega(t_{j+1}) - \omega(t_j)$, $0 \leqq j \leqq n - 1$, are independent and $\omega(t_{j+1}) - \omega(t_j)$ has distribution

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