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LOCAL NONDETERMINISM AND LOCAL TIMES OF GAUSSIAN PROCESSES[1]

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1. This work grew from a study of the conditions under which a Gaussian stochastic process has a "smooth" local time for almost all sample functions [**1**]–[**4**]. It is shown here that the main calculation in our previous work involves a property of Gaussian processes which is of independent interest—*local nondeterminism.* Let $X(t)$, $-\infty < t < \infty$, be a Gaussian process with mean 0, and J an open interval on the t-axis. Suppose that

(1) $E[X(t)]^2 > 0$ and $E[X(t) - X(s)]^2 > 0$, for all s and t in J.

For arbitrary $t_1 < \cdots < t_m$, where $t_j \in J$, form the ratio V_m of the conditional to the unconditional variance:

$$V_m = \frac{\operatorname{Var}[X(t_m) - X(t_{m-1})|X(t_1), \ldots, X(t_{m-1})]}{\operatorname{Var}[X(t_m) - X(t_{m-1})]}.$$

The numerator represents the error of prediction of $X(t_m) - X(t_{m-1})$ based on $X(t_1), \ldots, X(t_{m-1})$. X is called locally nondeterministic on J if

$$\lim_{c \downarrow 0} \inf_{t_m - t_1 \leqq c} V_m > 0, \text{ for every } m \geqq 2. \tag{2}$$

This is a local version of the classical notion of nondeterminism: it signifies that an observation is "relatively unpredictable" on the basis of a finite set of observations from the immediate past.

We find conditions under which the members of certain classes of Gaussian processes are locally nondeterministic: for example, processes of multiplicity 1, processes with stationary increments, and others.

2. Local nondeterminism means that there is an unremovable element of "noise" in the local evolution of the sample function. We expect such a function to be "locally irregular". And so it is: We show that local nondeterminism is one of the two main sufficient conditions in our result

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