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APPELL POLYNOMIALS WHOSE GENERATING FUNCTION IS MEROMORPHIC ON ITS CIRCLE OF CONVERGENCE

BY J. D. BUCKHOLTZ

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Let $\Phi(z) = \sum_0^\infty \beta_j z^j$ have radius of convergence r $(0 < r < \infty)$ and no singularities other than poles on the circle $|z| = r$. The Appell polynomials generated by Φ are given by

$$\pi_k(z) = \sum_{j=0}^{k} \beta_{k-j}\, z^j/j!, \qquad k = 0, 1, 2, \cdots.$$

An entire function g is said to possess a $\{\pi_k\}$ expansion if there is a complex sequence $\{h_k\}_0^\infty$ such that

$$\sum_{k=0}^{\infty} h_k \pi_k(z) \tag{1}$$

converges uniformly on compact sets to $g(z)$. In this note we show that the family of functions which have $\{\pi_k\}$ expansions is completely determined by the poles of Φ on $|z| = r$ together with the zeros of Φ in the closed disk $|z| \leqq r$.

Set $\Phi(z) = T(z)\phi_1(z)/P(z)$, where ϕ_1 is analytic and zero-free in $|z| \leqq r$ and T and P are polynomials whose zeros correspond respectively to the zeros of Φ in $|z| \leqq r$ and the poles of Φ on $|z| = r$. Let

$$P(z) = \prod_{q=1}^{\lambda} (1 - \alpha_q z)^{m(q)},$$

where $m(q)$ denotes the multiplicity of the pole α_q^{-1} of Φ, and let $m = \max m(q)$, $1 \leqq q \leqq \lambda$. It is relatively easy to characterize those complex sequences $\{h_k\}_0^\infty$ for which (1) converges. The following result was proved in [**2**], and can also be obtained as a special case of a theorem of W. T. Martin [**3**].

THEOREM A. *If $\{h_k\}_0^\infty$ is a complex sequence, then the following are equivalent*:

(i) *each of the series*

$$\sum_{k=0}^{\infty} \binom{k + m(q) - 1}{m(q) - 1} h_k \alpha_q^k, \qquad 1 \leqq q \leqq \lambda,$$

converges;

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