BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 2, March 1973

ON THE MULTIPLICITY OF THE SPECTRUM OF THE SPACE OF CUSP FORMS OF GL_n[1]

BY J. A. SHALIKA

Communicated by C. C. Moore, July 31, 1972

In this note I wish to announce certain preliminary results concerning a class of special functions arising in the theory of representations of groups defined over local fields. Before the preparation of this manuscript, the author obtained a copy of the recent paper [**4**] of I. M. Gelfand and D. A. Kajdan. Several of the results announced here are proved in their paper. I hope to indicate below what I have obtained independently and also what new results I have obtained since the paper of Gelfand and Kajdan became available. I wish to thank A. W. Knapp for the proof of a central lemma stated below.

1. **A conjecture.** Let k be a global field. Let $\boldsymbol{G}$ be an algebraic group defined over k. If R is a commutative k-algebra with identity, $\boldsymbol{G}_R$ will denote the group of points of $\boldsymbol{G}$ rational over R. The notation $\boldsymbol{G}(R)$ will also be used when convenient. $R^\times$ will denote the unit group of R. Let A denote the ring of adeles of k. Let ω be a (unitary) character of $A^\times$ trivial on $k^\times$. For $\boldsymbol{G} = GL_n$, let

$$C_\omega = {}^0L_2(\boldsymbol{G}_A/\boldsymbol{G}_k, \omega)$$

denote the space of cusp forms on $\boldsymbol{G}_A$ associated with ω. Let $\hat{\boldsymbol{G}}_A$ denote the set of equivalence classes of admissible, irreducible, unitary representations of $\boldsymbol{G}_A$. For $\boldsymbol{G} = GL_n$, $\Pi \in \hat{\boldsymbol{G}}_A$, let $m_0(\Pi, \omega)$ denote the multiplicity with which Π occurs in C_ω.[2]

In this note, I want to present some evidence in support of the following:

CONJECTURE 1. For $\Pi \in \widehat{GL_n}(A)$, $m_0(\Pi, \omega) \leqq 1$.

The conjecture has, of course, been proved for GL_2. Classically this statement, somewhat reformulated, appears in the well-known works of E. Hecke and H. Maass in their development of the theory of automorphic forms on GL_2. A systematic and general approach to Hecke theory was carried out in the framework of the theory of representations by H. Jacquet and R. P. Langlands [**11**]. The theory was also developed in a general setting by A. Weil [**18**]. In the work of Jacquet-Langlands, the proof of the simplicity of the spectrum of the space of cusp forms for GL_2 is reduced,

AMS (MOS) subject classifications (1970). Primary 22E55, 10D20; Secondary 33A75, 46F05.

[1] Research partially supported by NSF grant No. 31348.

[2] For $ch(k) = 0$, it is well known [**11**] that the assumption $m_0(\Pi, \omega) > 0$ implies that Π is admissible.

Copyright © American Mathematical Society 1973