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ON SUBSPACES OF SEPARABLE NORM IDEALS

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1. Let $\mathscr{H}$ be a separable infinite-dimensional Hilbert space and let $\mathscr{L}(\mathscr{H})$ denote the space of all bounded linear operators on $\mathscr{H}$. In the study of operators on Hilbert space and its many applications, particularly in mathematical physics, certain ideals of the ring $\mathscr{L}(\mathscr{H})$ are of fundamental importance. These are the ones called *norm ideals* by Schatten [**11**] and *s.n. ideals* (*symmetric norm ideals*) by Gohberg and Krein [**5**]. Examples of such ideals are the space $K(\mathscr{H})$ of compact operators, the space $N(\mathscr{H})$ of nuclear (or trace-class) operators, the Hilbert-Schmidt operators and their natural extensions, the classes C_p $(1 \leqq p < +\infty)$ [**3**], [**9**], and the rather recently introduced classes C_Ω^0 and C_ω of Gohberg and Krein [**5**], [**8**]. The last are already of considerable importance, arising naturally in the study of the abstract triangular integral and in questions concerning the completeness of the root vectors of an operator [**1**], [**6**], [**7**], [**8**], yet have barely been studied in any depth. In fact, though all of the ideals mentioned above are separable Banach spaces, almost no work concerning their subspace structure has been done (one exception is the fine study of the ideals C_p by McCarthy [**9**]).

The purpose of this paper is to begin a study of the subspaces of separable norm ideals (by subspace we always mean a *closed* linear submanifold). In particular, we study the spaces $K(\mathscr{H})$, $N(\mathscr{H})$, C_Ω^0, and C_ω from this standpoint. We refer the reader to [**5**] for the necessary background information and notation concerning s.n. ideals. Details of the proofs will appear elsewhere.

2. **The ideals $K(\mathscr{H})$ and C_Ω^0.** Though the ideals $K(\mathscr{H})$ and C_Ω^0 are dissimilar in many respects, their subspace structures are very much alike.

THEOREM 1. *Let E be a closed subspace of $K(\mathscr{H})$. Then either E is isomorphic to $\mathscr{H}$ or it contains a subspace isomorphic to c_0. If E is isomorphic to $\mathscr{H}$, then it is complemented in $\mathscr{L}(\mathscr{H})$.*

The idea of the proof of Theorem 1 is as follows: $K(\mathscr{H}) = \mathscr{H}^* \otimes_\alpha \mathscr{H}$ for a uniform crossnorm α (in fact $\alpha = \lambda$, the "least" crossnorm [**11**]). Moreover, if (ϕ_i) is an orthonormal basis for $\mathscr{H}$, then $(\phi_i \otimes \phi_j)$ is a basis

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