BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 2, March 1973

SINGULAR INTEGRALS AND ESTIMATES FOR THE CAUCHY-RIEMANN EQUATIONS

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Communicated August 15, 1972

1. **Introduction.** Our purpose here is to give new estimates for the inhomogeneous Cauchy-Riemann equations $\bar{\partial} u = f$, in a smooth strictly pseudo-convex domain in C^n. The estimates we shall present should not, however, be regarded as isolated calculations; they are part of a larger pattern of results which arise from adopting the following general point of view[1]: There is an intimate connection between some new singular integrals that arise (and the estimates to be made for them) in the following areas:

(1) In the context of nilpotent groups, motivated partly by the study of intertwining operators. See [**5**], [**8**], and [**1**], and §7 below.

(2) The singular integrals that occur in the solution of the $\bar{\partial} u = f$ problem, as given by Grauert and Lieb [**2**], Henkin [**3**], and Kerzman [**4**].

(3) Boundary behavior of holomorphic functions of several complex variables. See [**12**] and §5 below.

(4) Singular integrals related to the Bergman kernel. See §6.

From the qualitative point of view what is common to these areas (and distinguishes them from the more classical integrals and estimates) is the splitting of directions at each point, together with the nonisotropic way that singularities behave and estimates are made. The nonisotropy is not always apparent for L^p estimates, but its role becomes clear when calculating with the appropriate Lipschitz (or Hölder) inequalities. It is with the latter type of estimates that we are principally concerned here. Detailed proofs and further results will be given at another occasion.

2. **Standard Lipschitz spaces.** Recall the definition of $\Lambda_\alpha(\boldsymbol{R}), 0 < \alpha < \infty$, as given in [**11**, Chapter V]. From it we can define the corresponding spaces $\Lambda_\alpha(I)$, for the unit interval $I = [0, 1]$ in the usual way, as the Banach space obtained by restrictions, i.e., as the quotient $\Lambda_\alpha(\boldsymbol{R})/\mathscr{I}$, where $\mathscr{I} = \{f \in \Lambda_\alpha(\boldsymbol{R}) : f(t) = 0, t \in I\}$.

Next, suppose $\mathscr{R}$ is a smooth Riemannian manifold which for simplicity we assume is isometrically embedded in Euclidean $\boldsymbol{R}^N$. For each integer l we define the class of curves $\mathscr{C}^l$ in $\mathscr{R}$ to be $\mathscr{C}^l = \{x(t) : I \ni t \to x(t) \in \mathscr{R}, |x'(t)| \leqq 1, \ldots, |x^{(l)}(t)| \leqq 1\}$. We can now define $\Lambda_\alpha(\mathscr{R})$. Let l be smallest

AMS(MOS) subject classifications (1970). Primary 44A25, 35N15, 26A16; Secondary 43A80.

[1] Further background for this approach may be found in [**10**].