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PERIODIC AND HOMOGENEOUS STATES ON A VON NEUMANN ALGEBRA. II[1]

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This paper is a natural continuation of the previous paper [**9**]. In [**9**], we proved a structure theorem for a von Neumann algebra with a fixed periodic and homogeneous state. In this paper, we will show that the structure theorem in [**9**] determines intrinsically the algebraic type of a factor with a periodic and *inner* homogeneous state (see Definition 1). We keep the terminologies and the notations in [**9**].

DEFINITION 1. A normal state φ on a von Neumann algebra $\mathscr{M}$ is said to be *inner homogeneous* if $G(\varphi) \cap \mathrm{Int}(\mathscr{M})$ acts ergodically on $\mathscr{M}$, that is, if the group of all inner automorphisms of $\mathscr{M}$ leaving φ invariant has no fixed points other than the scalar multiples of the identity.

For each $a \in \mathscr{M}$, we write

$$\mathrm{Ad}(a)x = axa^*, \qquad x \in \mathscr{M}.$$

Since $\mathrm{Ad}(u) \in G(\varphi)$ for a unitary $u \in \mathscr{M}$ if and only if u falls in $\mathscr{M}_\varphi$, the centralizer of φ, the inner homogeneity of φ is equivalent to the fact that $\mathscr{M}'_\varphi \cap \mathscr{M} = \{\lambda 1\}$. Hence $\mathscr{M}_\varphi$ is a II_1-factor and $\mathscr{M}$ itself is also a factor.

We consider two periodic and inner homogeneous faithful normal states φ and ψ on $\mathscr{M}$. We denote by $\{\mathscr{M}_n^\varphi : n = 0, \pm 1, \ldots\}$ and $\{\mathscr{M}_n^\psi : n = 0, \pm 1, \ldots\}$ the decompositions of $\mathscr{M}$ in [**9**, Theorem 11] corresponding to φ and ψ respectively. By [**9**, Theorem 13], φ and ψ have the same period, say $T > 0$. Let $\kappa = e^{-2\pi/T}$, $0 < \kappa < 1$.

Following Connes' idea, we consider the tensor product $\mathscr{P} = \mathscr{M} \otimes \mathscr{L}(\mathfrak{H}_2)$ of $\mathscr{M}$ and the 2×2-matrix algebra $\mathscr{L}(\mathfrak{H}_2)$. Let $\{e_{i,j} : i, j = 1, 2\}$ be a system of matrix units in $\mathscr{L}(\mathfrak{H}_2)$. Every $x \in \mathscr{P}$ is of the form

$$x = x_{11} \otimes e_{11} + x_{12} \otimes e_{12} + x_{21} \otimes e_{21} + x_{22} \otimes e_{22},$$

where $x_{ij} \in \mathscr{M}$. We define a faithful state χ on $\mathscr{P}$ by

$$\chi(x) = \tfrac{1}{2}(\varphi(x_{11}) + \psi(x_{22})).$$

Connes showed in [**3**] that there exists a strongly continuous one-

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