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NORMAL FORMS FOR SYSTEMS OF SEMILINEAR HYPERBOLIC PARTIAL DIFFERENTIAL EQUATIONS

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1. **Introduction.** That a second order semilinear hyperbolic partial differential equation in two independent variables can be transformed into normal form so that differentiation is with respect to characteristic directions is well known. In this note we announce a generalization of this idea to systems of second-order semilinear hyperbolic equation in two variables. As an intermediate step a variational principle for the eigenvalues of a strongly-damped eigenparameter problem is developed [**1**]. Details will appear elsewhere.

2. **Definitions and notation.** (a) As usual an $n \times n$ Hermitian matrix P is positive definite if and only if the eigenvalues of P are strictly positive. We write $P > 0$.

(b) We define an inner product on $\boldsymbol{C}^n$ by

$$(x, y) = \sum_{i,j=1}^{n} x_i P_{ij} \bar{y}_j. \tag{2.1}$$

(c) If A, B, C are $n \times n$ matrices, define quadratic forms a, b, c by

$$a(x) = (Ax, x), \quad b(x) = (Bx, x), \quad c(x) = (Cx, x). \tag{2.2}$$

(d) The quadratic eigenparameter equation

$$L(\lambda)x = \lambda^2 Ax + \lambda Bx + Cx = 0 \tag{2.3}$$

is called strongly damped if

$$\begin{aligned} &\text{(i)} \quad PA > 0, \qquad \text{(ii)} \quad a, b, c \text{ are real valued,} \\ &\text{(iii)} \quad b^2(x) > 4a(x)c(x) \quad \text{for } x \neq 0. \end{aligned} \tag{2.4}$$

(e) The first order semilinear system of partial differential equations

$$v_t = Dv_x + h(x, t, v) \tag{2.5}$$

is in normal form if D is a diagonal matrix.

(f) A (single-valued) real valued function f defined on a real interval I is real analytic on I if and only if f has a convergent Taylor's series expansion about each point of I always with nontrivial interval of convergence.

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