BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 2, March 1973

# WEIGHTED APPROXIMATION FOR MODULES OF CONTINUOUS FUNCTIONS

BY W. H. SUMMERS[1]

Communicated by François Treves, September 25, 1972

Nachbin [**6**] has enjoyed notable success with the weighted approximation problem in both the real and selfadjoint complex cases, but little progress has yet been realized in the general complex case, even though interest dates from the problem's formative stages (cf. [**4**, p. 1057], [**5**, p. 126]). The purpose of the present note is to provide an answer to this question in a setting which occupies a pivotal position in the theory developed by Nachbin; namely, the bounded case of the weighted approximation problem.

1. **Preliminaries.** In what follows, all functions will be assumed complex valued unless explicitly stated otherwise, while $X$ will denote a completely regular Hausdorff space, $C(X)$ will denote the algebra of all continuous functions on $X$, and $B_0(X)$ will denote the algebra of all bounded functions on $X$ which also vanish at infinity. We will assume that $B_0(X)$ is equipped with the uniform (convergence) topology induced by $\|\cdot\|$, the usual supremum norm defined on the bounded functions on $X$.

We now introduce a set $V$ of nonnegative upper semicontinuous functions on $X$; the elements of $V$ being referred to as *weights*. The corresponding *weighted space* $CV_0(X)$ is the locally convex topological vector space obtained by equipping the linear subspace consisting of those $f \in C(X)$ such that $fv \in B_0(X)$ for every $v \in V$ with the *weighted topology* $\omega_V$ generated by the seminorms $p_v$, one for each $v \in V$, defined on this space by $p_v(f) = \|fv\|$. Since there is no loss of generality in so doing, we will assume that if $u, v \in V$ and $\lambda \geqq 0$, then there is a $w \in V$ for which $\lambda u, \lambda v \leqq w$ (pointwise); i.e., $V$ is a Nachbin family on $X$ [**8**, p. 90]. In addition, we henceforth assume that a subalgebra $A$ of $C(X)$ and a linear subspace $W$ of $CV_0(X)$, where $W$ is an $A$-module with respect to pointwise multiplication, have been specified. The *weighted approximation problem* [**6**, p. 293] asks for a description of the closure of $W$ in $CV_0(X)$.

2. **The bounded case of the weighted approximation problem.** The setting Nachbin termed the *bounded case* of the weighted approximation problem [**6**, p. 294] is the one in which every $a \in A$ is bounded on the support of each $v \in V$, and it is in this situation that a meaningful characterization of

---

*AMS(MOS) subject classifications* (1970). Primary 41A65, 46E10.
[1] This research was supported in part by National Science Foundation Grant GP-34370.