BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 2, March 1973

SPECTRAL MAPPING THEOREMS ON A TENSOR PRODUCT

BY ROBIN HARTE

Communicated by Robert G. Bartle, August 17, 1972

1. **Introduction.** By computing the joint spectrum [**5**], [**6**] for certain systems of elements in a tensor product [**3**], [**11**] of Banach algebras, and applying the spectral mapping theorem in several variables [**5**], [**6**], [**7**], we find that we can determine the spectrum of certain linear operators, notably the tensor product $S \otimes T$ discussed by Brown and Pearcy [**1**], [**12**]. We can also see that the spectrum of an "operator matrix" [**4**], [**10**] is what it ought to be, and recover the results of Lumer and Rosenblum [**10**] about the multiplication operators $L_S R_T$ and $L_S + R_T$. Full proofs, and more detail, will appear elsewhere [**8**].

2. **Left and right spectra.** Suppose that A is a complex Banach algebra, with identity 1. Then the *joint spectrum* of a system of elements $a \in A^n$ is the union of the *left spectrum* and the *right spectrum* [**5**, Definition 1.1]:

$$\sigma_A^{\text{joint}}(a) = \sigma_A^{\text{left}}(a) \cup \sigma_A^{\text{right}}(a) \tag{2.1}$$

where

$$\sigma_A^{\text{left}}(a) = \left\{ s \in C^n : 1 \notin \sum_{j=1}^{n} A(a_j - s_j) \right\} \tag{2.2}$$

and

$$\sigma_A^{\text{right}}(a) = \left\{ s \in C^n : 1 \notin \sum_{j=1}^{n} (a_j - s_j)A \right\}. \tag{2.3}$$

The *spectral mapping theorem* [**5**, Theorem 3.2] is the equality

$$\sigma_A^{\text{joint}} f(a) = f \sigma_A^{\text{joint}}(a), \tag{2.4}$$

valid for a commuting system of elements $a \in A^n$ and a system $f = (f_1, f_2, \ldots, f_m)$ of polynomials in n complex variables. Equality (2.4) is also valid for left and right spectra separately; it extends [**7**, Theorem 4.2] to certain noncommuting systems of elements, where of course the idea of a "polynomial" has to be extended. Here we take a "polynomial in n variables" to be an element of the free complex algebra-with-identity Poly_n on n generators z_j; for an arbitrary system of elements $a \in A^n$, the mapping $f \to f(a): \text{Poly}_n \to A$ is a homomorphism which preserves

---

AMS (*MOS*) *subject classifications* (1970). Primary 47D99, 46H99; Secondary 47A10, 46M05.
Key words and phrases. Joint spectrum, spectral mapping theorem, tensor product, uniform crossnorm, operator matrix, multiplication operator.