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DOUBLE CENTRALIZERS OF PEDERSEN'S IDEAL OF A C^*-ALGEBRA. II

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1. **Introduction.** This note is a sequel of [**1**], to which we refer for motivation and basic terminology. In this note we develop a comprehensive spectral theory and functional calculus for the double centralizers of Pedersen's ideal. The full details of our discussion will appear elsewhere, and we intend to pursue related topics in subsequent papers.

From now on A will denote a C^*-algebra, K_A its Pedersen ideal (or simply K if A is understood), and $\Gamma(K)$ the double centralizers of K. As in [**1**] we will view the double centralizers of A, denoted by $\Gamma(A)$ or $M(A)$, and A as subalgebras of $\Gamma(K)$.

2. **Spectral theory and a functional calculus.** For each $a \in K$ let $\mathscr{L}_a$ and $\mathscr{R}_{a^*}$ denote the closed left and right ideals of A generated by a and a^* respectively. Note that $\mathscr{L}_a$ and $\mathscr{R}_{a^*}$ are subsets of K (see [**1**]). Now let Γ_a denote the set of all pairs (U, V) that satisfy the following: (i) U and V are bounded linear operators on $\mathscr{L}_a$ and $\mathscr{R}_{a^*}$ respectively; (ii) $xU(y) = V(x)y$ for each $x \in \mathscr{R}_{a^*}$ and $y \in \mathscr{L}_a$.

Let (S, T) and (U, V) belong to Γ_a and let α be a complex number. Then it is clear that $(S + U, T + V)$, $(\alpha U, \alpha V)$, and (SU, VT) belong to Γ_a. Moreover, if we define S^* on $\mathscr{R}_{a^*}$ by the formula $S^*(x) = S(x^*)^*$ and similarly define T^* on $\mathscr{L}_a$, then (T^*, S^*) belongs to Γ_a. Consequently, Γ_a is a $*$-algebra when provided with the following operations: (i) $(S, T) + (U, V) = (S + U, T + V)$; (ii) $\alpha(S, T) = (\alpha S, \alpha T)$; (iii) $(S, T)(U, V) = (SU, VT)$; (iv) $(S, T)^* = (T^*, S^*)$.

PROPOSITION 2.1. *If* $(S, T) \in \Gamma_a$, *then* $\|S\|^2 = \|S^*\|^2 = \|T\|^2 = \|T^*S\|$. *Consequently, the $*$-algebra Γ_a provided with the norm* $\|(S, T)\| = \|S\|$ *is a C^*-algebra with identity.*

For each double centralizer (S, T) of K define $\lambda_a(S) = S|\mathscr{L}_a$, $\rho_a(T) = T|\mathscr{R}_{a^*}$, and $\phi_a((S, T)) = (\lambda_a(S), \rho_a(T))$. Since $\mathscr{L}_a$ and $\mathscr{R}_{a^*}$ are invariant under S and T respectively, ϕ_a is a well-defined map of $\Gamma(K)$ into Γ_a. Moreover, it is clear that ϕ_a is a $*$-homomorphism.

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