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NONUNIQUE CONTINUATION FOR UNIFORMLY PARABOLIC AND ELLIPTIC EQUATIONS IN SELFADJOINT DIVERGENCE FORM WITH HÖLDER CONTINUOUS COEFFICIENTS[1]

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Consider the problem of backward uniqueness for the uniformly parabolic equation

$$(1)\quad \begin{aligned} &(a) \qquad u_t = \sum_{i,j=1}^{n} (a_{ij}(x,t)u_{x_j})_{x_i} \equiv \nabla \cdot \mathscr{A}\nabla u \quad \text{in } \Omega \times [0,\infty), \\ &(b) \qquad \mathscr{A}\nabla u \cdot \nu = 0 \quad \text{on } \partial\Omega \times [0,\infty), \end{aligned}$$

and the problem of unique continuation (and uniqueness for the Cauchy problem) for the uniformly elliptic equation

$$(2)\qquad \sum_{i,j=1}^{n} (a_{ij}(x)u_{x_j})_{x_i} \equiv \nabla \cdot \mathscr{A}\nabla u = 0 \quad \text{in } \Omega,$$

where Ω is a bounded domain in R^n, ν denotes the unit normal to $\partial\Omega$, and the symmetric matrix $\mathscr{A}$ has its eigenvalues in $[\alpha, \alpha^{-1}]$, with $\alpha > 0$. We construct examples of nonuniqueness for (1) when $n = 2$, and for (2) when $n = 3$; in each case α may be arbitrarily close to 1 and the coefficients are also Hölder continuous.

Backward uniqueness for (1) with $\mathscr{C}^1$ coefficients was shown by Lions-Malgrange [**5**]; probably the simplest proof is that of Agmon-Nirenberg [**2**] and Agmon [**1**] using the general method of logarithmic convexity. Carleman [**4**] long ago established unique continuation for (2) with $\mathscr{C}^2$ coefficients when $n = 2$. For $n \geqq 3$, unique continuation for (2) with $\mathscr{C}^{2,1}$ coefficients was proved by Aronszajn [**3**], and more simply with $\mathscr{C}^1$ coefficients by Agmon [**2**]. See [**1**] and [**6**] for references to other results by Holmgren, Cordes, Hörmander, Landis, Lees and Protter, Bers and Nirenberg, and others.

An example of nonunique continuation was constructed by Plis [**6**] for a uniformly elliptic equation in the nondivergence form

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