

BEHAVIOR AT THE BOUNDARY OF A SOLUTION TO PLATEAU'S PROBLEM¹

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Knowledge of the existence and structure of tangent cones plays a basic role in the study of the interior structure of area minimizing integral currents [6, §5.4] and in the study of the structure of rectifiable varifolds [2]. Moreover, in his study of boundary regularity of area minimizing currents with smooth boundary, William Allard [1] makes extensive use of tangent cones at points on the boundary. It is therefore to be expected that tangent cones will continue to play a central role in the investigation of the behavior at the boundary of area minimizing currents. Here we present results concerning the existence and structure of oriented tangent cones at points on the boundary of an area minimizing integral current. The proofs will appear elsewhere. Terminology and notation will be consistent with that of [6]; see in particular the List of Notations on pp. 670 and 671.

I am indebted to William Allard for many stimulating discussions on this subject, especially with regard to Theorem 1, which he originally proved under stronger hypotheses.

Our results will be given in terms of the potential theoretic function $V^{\partial S}$ which was first considered by Radon [9] and subsequently was used by J. Král (in the case where $k = n$ and S is obtained by integration over a Borel subset of \mathbf{R}^n) in discussing the Neumann boundary value problem and the heat equation for domains with nonsmooth boundary [7], [8]. In [3] we extended the definition of $V^{\partial S}$ to the case where k is arbitrary and ∂S is replaced by a flat current T , and investigated the properties of V^T . Let B be an oriented $(n - k + 1)$ -dimensional linear subspace of \mathbf{R}^n , $T \in F_{k-1}(\mathbf{R}^n)$, $a \in \mathbf{R}^n$ and $r > 0$. We define

$$V^T(a, r) = n\alpha(n) \int_{SO(n)} M[T[B(a, r) \cap (a + g_{\#}B)] d\Psi g$$

where Ψ is the Haar measure on $SO(n)$ and $\alpha(n)$ is the volume of the unit ball in \mathbf{R}^n . In case $M(T) < \infty$, $V^T(a, r)$ is proportional to

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