BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 2, March 1973

CONTRIBUTION TO THE THEORY OF EULER'S FUNCTION $\varphi(x)$[1]

BY EMIL GROSSWALD

Communicated by Dock S. Rim, August 21, 1972

1. **Introduction.** The last few years have witnessed a renewed interest in the study of the number $N(n)$ of solutions of the equation

$$\varphi(x) = n, \tag{1}$$

where $\varphi(x)$ is Euler's totient function.

The purpose of the present paper is to give a sharpened (and corrected) version of a theorem of Carmichael (Theorem 1; see [**1**, Theorem II]) and the proof of a weak form of the

CONJECTURE. For all natural integers n, $N(n) \neq 1$.

Lower case letters (with or without subscripts, or superscripts) stand, in general, for natural integers, p and q, in particular, for odd rational primes.

2. **Main results.**

DEFINITION. The natural integer k is said to be *admissible*, if its (unique) representation as a sum of distinct powers of 2,

$$k = 2^{s_1} + 2^{s_2} + \cdots + 2^{s_r}, \qquad s_1 > s_2 > \cdots > s_r \geqq 0,$$

is such that $2^{2^{s_j}} + 1$ is a (Fermat) prime for each $j = 1, 2, \ldots, r$. The set of admissible integers is denoted by K.

REMARK. For $r = 0$ it is convenient to consider the corresponding $k = 0$ as an admissible integer; one observes that formally one has $2^0 + 1 = 2$, a prime.

THEOREM 1. *Let $\chi(k)$ be the characteristic function of the set K ($\chi(k) = 1$ if $k \in K$, $\chi(k) = 0$ if $k \notin K$) and set $g(m) = \sum_{0 \leqq k \leqq m} \chi(k)$; then, if $n = 2^m$, equation* (1) *has*

$$N(n) = g(m) + \chi(m) \tag{I}$$

solutions.

COROLLARY 1. *For $n = 2^m$, $N(2^m) =$* $\min(m + 2, 32)$.

AMS(MOS) subject classifications (1970). Primary 10A20; Secondary 10B15, 10B99, 10-01.

[1] This paper was written with partial support from the National Science Foundation, through the grant GP 23170.