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A UNIFIED TREATMENT OF SOME BASIC PROBLEMS IN HOMOTOPY THEORY[1]

BY PETER I. BOOTH

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0. **Introduction.** This note announces some new methods in algebraic topology, based on the results of [**1**], [**2**] and [**3**]. The relative lifting problem [**5**, p. 415] is fundamental to that subject. It includes the extension problem, the retraction problem, the lifting problem, the section problem, the relative section problem and the computation of $[X, Y]$ problem, among its particular cases. These problems (excluding the last one) are usually taken to concern just the existence of extensions, retractions, etc.; we use the terms in the wider sense, to include both the existence question and the homotopy classification question, for the respective extensions, retractions, etc. We will:

(i) prove that many cases of the above problems (including the computation of the absolute and relative cohomology groups of the total space of a fibration, and the computation of certain homotopy groups) are equivalent to "parallel problems", involving the restricted fibered mapping projection $(pq; a)$;

(ii) give some examples of solutions of parallel problems. Our solutions include one lifting problem and two extension problems. The extension results have some immediate consequences; they give new derivations for the exact cohomology sequences of Serre and Wang. Gysin's sequence is derived separately. Future papers will discuss these and other applications in detail.

Our argument is valid in several convenient categories, including the category of $\mathfrak{k}$-spaces [**4**], [**2**], [**7**] and the category of quasi-topological spaces [**6**], [**1**], [**2**]. This category of $\mathfrak{k}$-spaces, for a definition see [**2**, p. 276], contains the usual category of Hausdorff k-spaces ($=$ compactly generated spaces $=$ Kelley spaces) as a subcategory.

1. **Preliminaries.** $\approx$ will be used to denote bijections and isomorphisms; $\cong$ to denote homeomorphisms. We outline the concepts from [**1**], [**2**] and [**3**] that are required. If X and Y are spaces, then $\mathscr{M}(X, Y)$ will denote the space of maps of X into Y; if $f \in \mathscr{M}(X, Y)$ then $\mathscr{M}(X, Y; f)$ will denote the path component of f with the subspace topology.

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