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SOME RESULTS ON THE CENTER OF A RING WITH POLYNOMIAL IDENTITY

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Introduction. The purpose of this paper is to provide a fresh outlook to various questions on rings with polynomial identity by examining the centers of such rings. This approach yields the interesting result that any nonzero ideal of a semiprime ring with polynomial identity intersects the center nontrivially (Theorem 2).

There are at least two interesting consequences to Theorem 2: a generalization of Wedderburn's theorem (any semiprimitive ring with polynomial identity, whose center is a field, is simple) and a strengthening of Posner's theorem [**1**] (any prime ring with a polynomial identity has a simple ring of quotients whose center is the quotient field of the center of the prime ring).

The proofs are elementary modulo Jacobson [**3**]. Of course rings are not necessarily commutative and for the sake of simplicity we assume a unit 1.

The key argument in this paper is an application of Formanek's central polynomials for matrix algebras over a field, whose important properties are [**2**]: Let M_n be an $n \times n$ matrix algebra over an arbitrary field. Then there exists a polynomial $g_n(X_1, \ldots, X_m)$ which has coefficients in $\boldsymbol{Z}$; is homogeneous (degree > 0) in every variable and linear in all but the first variable; takes values in the center for every specialization in M_n; and is nonvanishing for some specialization.

LEMMA 1. *$g_n(X_1, \ldots, X_m)$ is central, nonvanishing for any central simple algebra S of degree n over its center C.*

PROOF. Let us first consider C finite. Since by Wedderburn's structure theorem S is a matrix algebra over a division ring D which is finite dimensional over C, which is finite, we have D is finite and thus a field (Wedderburn's theorem on finite division rings [**3**, p. 183]). Thus $D = C$ and S is in fact a matrix algebra over C, a field, and g_n is by hypothesis a central, nonvanishing polynomial for S, so that there is nothing to prove.

So we may assume C is infinite. Again let S be a matrix algebra over D, a division ring finite dimensional over C. Let F be a splitting subfield

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