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FREE FINITE GROUP ACTIONS ON S^3

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In this paper we describe the first stages of a theory of 3-manifolds with finite fundamental group. The strong conjecture that any free finite group action on S^3 is conjugate to a linear action is known for some cyclic groups, see [**3**], [**4**], and is supported by recent work of one of us on fundamental groups [**2**]. Here we concern ourselves with the weaker conjecture that any compact 3-manifold with finite fundamental group is homotopy equivalent to a Clifford-Klein form. (Note that both conjectures are phrased to avoid problems with homotopy 3-spheres.) It is known, see for example [**6**], that the homotopy type of such a manifold is determined by the fundamental group and the first k-invariant. By exploiting the link between k-invariant and finiteness obstruction we are able to decide which homotopy types correspond to *finite* Poincaré complexes, and thus restrict the possible homotopy types for manifolds. There are nonstandard types for some groups, and a corollary of our argument is the existence in dimensions $4n - 1$, $n \geqq 2$, of free actions homotopically distinct from orthogonal ones. When $n = 1$, we can only produce such an action on a homology sphere, and it would be most interesting to know the fundamental group.

1. **Homotopy type of space forms.** Let the abstract group π be isomorphic to the fundamental group of a compact 3-dimensional manifold of constant positive curvature (Clifford-Klein form), and suppose π cannot be decomposed as a direct product. The possibilities for π are listed in the following table, see [**10**, Chapter 7, p. 224]:

If Y is a 3-dimensional CW-complex such that $\tilde{Y}$ is homotopy equivalent to S^3, and we can choose an isomorphism $\psi:\pi_1(Y, y) \to \pi$, we shall call Y a *Poincaré space form.* Y is not necessarily finite, and the isomorphism ψ, although not natural, is assumed fixed. Homotopy classes of space forms are in (1-1) correspondence with the orbits in $H^4(\pi, \boldsymbol{Z})$ under the action of $\pm \operatorname{Aut} \pi$ [**6**, Theorem 1.8] and there is a well-defined obstruction to finding a finite complex in a given homotopy type, lying in the projective class group $\tilde{K}_0(\boldsymbol{Z}\pi)$ [**8**, Theorem F]. We can describe this obstruc-

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