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PERIODIC AND HOMOGENEOUS STATES ON A VON NEUMANN ALGEBRA. I[1]

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This paper is devoted to announcing a structure theorem for von Neumann algebras admitting a periodic homogeneous faithful state (see Definitions 1 and 2).

Let $\mathscr{M}$ be a von Neumann algebra. Suppose that ϕ is a faithful normal state on $\mathscr{M}$. We denote by σ_t^ϕ the modular automorphism group of $\mathscr{M}$ associated with ϕ. Let $G(\phi)$ denote the group of all automorphisms of $\mathscr{M}$ which leave ϕ invariant. We introduce the following terminologies concerning ϕ.

DEFINITION 1. If there exists $T > 0$ such that σ_T^ϕ is the identity automorphism of $\mathscr{M}$, denoted by ι, then we call ϕ *periodic*. The smallest such number T is called the *period* of ϕ.

DEFINITION 2. We call ϕ *homogeneous* if $G(\phi)$ acts ergodically on $\mathscr{M}$; that is, the fixed points of $G(\phi)$ are only scalar multiples of the identity.

DEFINITION 3. We call ϕ *ergodic* if $\{\sigma_t^\phi\}$ is ergodic.

The ergodicity of ϕ implies the homogeneity of ϕ, since $\{\sigma_t^\phi\}$ is contained in $G(\phi)$. Furthermore, if $\mathscr{M}$ admits an ergodic state, then $\mathscr{M}$ must be a factor.

Now, suppose ϕ is a periodic homogeneous faithful normal state on $\mathscr{M}$, which will be fixed throughout the discussion. Considering the cyclic representation of $\mathscr{M}$ induced by ϕ, we assume that $\mathscr{M}$ acts on a Hilbert space $\mathfrak{H}$ with a distinguished cyclic vector ξ_0 such that $\phi(x) = (x\xi_0|\xi_0)$, $x \in \mathscr{M}$. According to the theory of modular Hilbert algebras (which the author proposes to call Tomita algebras), there exists the positive self-adjoint operator Δ on $\mathfrak{H}$ and the unitary involution J on $\mathfrak{H}$ such that

$$\sigma_t^\phi(x) = \Delta^{it}x\Delta^{-it}, \qquad x \in \mathscr{M};$$

$$\Delta^{it}\xi_0 = \xi_0;$$

$$J\mathscr{M}J = \mathscr{M}'; \qquad J\Delta^{it}J = \Delta^{it}.$$

Put $\alpha = e^{-2\pi/T}$ with T the period of ϕ. Obviously, we have $0 < \alpha < 1$. We introduce the following notations:

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