

A CLASS OF π_c GROUPS CLOSED UNDER CYCLIC AMALGAMATIONS

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This note is to announce the existence of a class \mathcal{C} of π_c groups (see below for definition of π_c) that is closed under the generalized free product with a single cyclic subgroup amalgamated. The class \mathcal{C} has the additional property that if $A \in \mathcal{C}$ and B is any π_c group then the generalized free product $G = *(A, B; a_0 = b_0)$, where a_0 and b_0 generate isomorphic subgroups of A and B respectively, is again a π_c group. (However, it may be that $G \notin \mathcal{C}$.) In contrast we show that for each residually finite group A with an element a_0 of infinite order, there is a residually finite group B and an element b_0 in B such that the generalized free product $*(A, B; a_0 = b_0)$ is not residually finite.

The theorems above provide new proofs (as well as important generalizations) of previous theorems of P. Stebe [2] and G. Baumslag [1]. Details will appear elsewhere.

Let \mathcal{C} denote the class of all π_c groups A with the property that if B is any π_c group, then the generalized free product $*(A, B; a_0 = b_0)$ is a π_c group. Recall that a group G is a π_c group (as defined in [2]) if, for every pair of elements g_1, g_2 of G , either $g_1 \in \langle g_2 \rangle$ or there exists a normal subgroup N of G having finite index with $\bar{g}_1 \notin \langle \bar{g}_2 \rangle \nu$ in G/N (bar denotes coset modulo N).

THEOREM 1. *If A and B are both in \mathcal{C} , and if a_0 and b_0 generate isomorphic subgroups of A and B respectively, then $*(A, B; a_0 = b_0)$ is also in the class \mathcal{C} .*

The proof of Theorem 1 requires a study of the finite quotient groups of π_c groups. With each element g of a group G , we associate a set $G(g)$ of positive integers with the property that $n \in G(g)$ if and only if G has a finite quotient group in which the image of g has order n . Let g be a non-trivial element of a group G . A subset X of $G(g)$ is said to be *cofinal* in $G(g)$ if for each pair $g_1, g_2 \in G$ ($g_1 \neq 1$), either $g_1 = g_2^t$ for some t or there is a homomorphism f of G onto a finite group such that $f(g_1) \neq f(g_2)^t$ for any t , and the order of $f(g)$ is in X . In particular G is a π_c group if $G(g)$ is cofinal in $G(g)$ for some g in G . More generally, we can prove the following lemma.

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