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POSITIVE HARMONIC FUNCTIONS AND BIHARMONIC DEGENERACY[1]

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The class O_{HP} of Riemann surfaces or Riemannian manifolds which do not carry (nonconstant) positive harmonic functions is the smallest harmonically or analytically degenerate class. In particular, it is strictly contained in the classes O_{HB} and O_{HD} of Riemann surfaces or Riemannian manifolds without bounded or Dirichlet finite harmonic functions, and in the classes O_{AB} and O_{AD} of Riemann surfaces without bounded or Dirichlet finite analytic functions.

In the present paper we ask: Are there any relations between O_{HP} and the classes O_{H^2B} and O_{H^2D} of Riemannian manifolds without bounded or Dirichlet finite nonharmonic biharmonic functions? We shall show that the answer is in the negative. Explicitly, if O^N is a null class of N-dimensional manifolds, and $\tilde{O}^N$ its complement, then all four classes

$$O^N_{HP} \cap O^N_{H^2X}, \qquad O^N_{HP} \cap \tilde{O}^N_{H^2X}, \qquad \tilde{O}^N_{HP} \cap O^N_{H^2X}, \qquad \tilde{O}^N_{HP} \cap \tilde{O}^N_{H^2X}$$

are nonempty for both $X = B$ and D, and for any N. This independence of N is of interest, as biharmonic degeneracy often fails to have this property. Typically, whereas the punctured Euclidean N-space is not an element of $O^N_{H^2B}$ for $N = 2, 3$, it does belong to it for all $N \geqq 4$ (Sario-Wang [**6**]).

Methodologically, we introduce in §1 a simple type of Riemannian manifold which, on account of its rectangular coordinates and nonconformal metric, is very versatile in classification problems.

1. We shall show

THEOREM 1. *$O^N_{HP} \cap \tilde{O}^N_{H^2B} \neq \emptyset$ for every N.*

PROOF. Consider the N-manifold, $N \geqq 2$,

$$T = \{0 < x < \infty, 0 \leqq y \leqq 2\pi, 0 \leqq z_i \leqq 2\pi\},$$

$i = 1, \ldots, N - 2$, with $y = 0$, $y = 2\pi$ identified, and $z_i = 0$, $z_i = 2\pi$ also identified for every i. Endow T with the metric

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