BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 1, January 1973

COMBINATORIAL SYMMETRIES OF THE m-DISC. I

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Communicated by Michael Atiyah, May 30, 1972

In 1938–1939 P. A. Smith published this theorem about combinatorial symmetries of the m-disc having prime period p: If K denotes the fixed point set of $Z_p \times D^m \to D^m$, then K is a Z_p-homology manifold and Z_p-homology disc (see [4]).

The main result in this paper states that for odd primes p, there is a homogeneity property of the fixed point set K which is not implied by P. A. Smith's theorem: There is a characteristic class $h^p_{4*+K-1} \in H_{4*+K-1}(K/\partial K, Z_2)$, which must vanish if K is to be the fixed point set for some $Z_p \times D^m \to D^m$, where K denotes both the polyhedron K and its dimension. By homogeneity it is meant that the class vanishes if K is a PL manifold (see Theorem 1.1b). Theorem 1.2 is intended to clarify the mechanics that define h^p_{4*+K-1}: represent homology classes by singular Z_p-homology manifolds P_i; compute an invariant from the mid-dimensional intersection forms of the P_i; use the universal coefficient theorem to get h^p_{4*+K-1}. This is a procedure well known to workers in the field (see [3], [6]).

A key step in determining the properties of this characteristic class, relates an exponent 4 invariant of the fixed point set for $Z_p \times M \to M$, to the Z_p-index of M, where $Z_p \times M \to M$ is a combinatorial symmetry on a closed PL manifold M.

Results are stated only for primes of the form $p = 4q + 1$ with $q =$ odd. Similar results hold for other odd primes, but tables and invariants must be slightly modified.

In part, this is a correction to [1]. There, in a remark, it is said that the converse to P. A. Smith's theorem is true for odd primes, provided the potential fixed point set admits a "2-parameter cross section." This is not true, as Theorems 1.1, 1.3 below show.

1. **Characteristic classes measuring nonhomogeneity.** $(K, \partial K)$ denotes a Z_p-homology manifold pair, σ denotes any exponent 4 invariant that can be additively associated to quadratic forms over the integers having determinant prime to p; e.g., various combinations of Hasse symbol and discriminant type invariants; reduction mod p invariants.

THEOREM 1.1. *There is a characteristic class* $h^\sigma_{4*+K-1} \in H_{4*+K-1}(K/\partial K, Z_4)$, *satisfying*

AMS (MOS) subject classifications (1970). Primary 57D20.