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SPACES OF EQUIVARIANT SELF-EQUIVALENCES OF SPHERES

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ABSTRACT. Let $F(S^m)$ denote the identity component of the space of homotopy self-equivalences of S^m and let $F = \text{inj}\lim_m F(S^m)$. This paper studies the homotopy properties of certain equivariant analogs of the infinite loop space F.

1. **Introduction.** Let G be a compact Lie group and let W be a free, finite dimensional, real G-module equipped with a G-invariant metric. Let $S(W)$ be the unit sphere of W and denote by $F(W)$ the identity component of the space of equivariant self-equivalences of $S(W)$ with the compact-open topology.

If V and W are free G-modules as above, then $V \oplus W$ is also a free G-module. Since $S(V \oplus W)$ is equivariantly homeomorphic to the join of $S(V)$ and $S(W)$, there is a continuous inclusion of $F(V)$ into $F(V \oplus W)$ defined by taking joins with the identity on $S(W)$. In particular, if kW denotes the direct sum of k copies of W, there is an inclusion of $F(kW)$ in $F((k+1)W)$. Define

$$F_G = \text{inj}\lim_k F(kW). \tag{1.1}$$

If G is the trivial group then $F_G = F$ is a familiar and widely studied object. An important aspect of this space is the existence of two infinite loop space structures, one induced by composition multiplication, the other induced by a canonical homotopy equivalence from F to the identity component of inj $\lim_m \Omega^m(S^m)$. One can show that F_G also has an infinite loop space structure induced by composition multiplication. Our results generalize to F_G the second infinite loop space structure on F.

Let BG denote a classifying space for G, let $\mathfrak{g}$ be the Lie algebra of G and let G act on $\mathfrak{g}$ via the adjoint representation. The balanced product of EG and $\mathfrak{g}$ is a vector bundle over BG that we shall call ζ. Let BG^ζ denote its Thom space.

THEOREM 1. *On the category of connected finite* CW-*complexes there is a natural equivalence of homotopy functors*

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