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RELATIVELY INVARIANT SYSTEMS AND THE SPECTRAL MAPPING THEOREM

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1. **Introduction.** In this note we consider the extension of the spectral mapping theorem ([**2**], [**3**]) to certain noncommuting systems of elements, notably the 'quasi-commuting' systems of McCoy [**5**]. Full proofs and more detail are to appear elsewhere [**4**].

2. **Relative joint spectra.** Suppose $a = (a_1, a_2, \ldots, a_n)$ is a system of elements in a complex Banach algebra A, with identity 1: then the *joint spectrum* of a with respect to A is ([**2**]; [**3**, Definition 1.1]) the set $\sigma(a) = \sigma_A^{\text{joint}}(a)$ of those systems $s = (s_1, s_2, \ldots, s_n)$ of complex numbers for which the system $a - s = (a_1 - s_1, a_2 - s_2, \ldots, a_n - s_n)$ generates a proper left, or proper right, ideal in A. The 'one-way' spectral mapping theorem ([**2**]; [**3**, Theorem 3.2]) is the inclusion

$$f\sigma(a) \subseteq \sigma f(a), \tag{2.1}$$

valid for an arbitrary system $a \in A^n$ of elements and an arbitrary system $f = (f_1, f_2, \ldots, f_m): A^n \to A^m$ of 'polynomials' in several variables on A. Equality

$$\sigma f(a) = f\sigma(a) \tag{2.2}$$

is attained [**3**, Corollary 3.3] if the system of polynomials has a 'left inverse' $g: A^m \to A^n$ for which $g(f(a)) = a$, or alternatively if the system of elements is commutative ([**2**]; [**3**, Theorem 4.3]). This second case is our 'spectral mapping theorem', of which we here consider the extension.

DEFINITION 1. *The joint spectrum of $b \in A^m$ relative to $a \in A^n$ in A is the set*

$$\sigma_{a=a}(b) = \{t \in \sigma(b) : \exists s \in \sigma(a), (s, t) \in \sigma(a, b)\}. \tag{2.3}$$

The idea is to offer a measurement of the failure of equality in (2.1); for example there is equality

$$\sigma_{f(a)=f(a)}(a) = \sigma(a) \tag{2.4}$$

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