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INVARIANT SUBSPACES OF L^∞ AND H^∞

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Let T be the unit circle, and let L^∞ and H^∞ be the usual spaces of bounded functions. Let R be the group of rotations $z \mapsto e^{i\lambda}z$ and let M be the Möbius group

$$z \mapsto e^{i\lambda}\frac{z - z_0}{1 - \bar{z}_0 z}\cdot$$

Let R and M act on L^∞ by substitution.

THEOREM 1. *Let F be a closed M-invariant subspace of L^∞, with $z \in F$ and $zF \subseteq F$. Then F does not properly contain any closed M-invariant subspaces of finite codimension.*

Examples of such subspaces F are $F = L^\infty$, $F = H^\infty$, $F = A$, $F = C(T)$, $F = \mathscr{R}$, and $F = \mathscr{R}_h$. Here, A is the disc algebra, $\mathscr{R}$ is the space of functions in H^∞ that have radial limits along every radius, and $\mathscr{R}_h$ is the space of functions in H^∞ for which the radial limit fails to exist at most on a set of $e^{i\theta}$ of Hausdorff h-measure 0.

THEOREM 2. *There exists an R-invariant closed hyperplane in L^∞ that contains the space $C(T)$ but does not contain H^∞.*

COROLLARY. *There exists an R-invariant closed hyperplane in H^∞ that contains A.*

THEOREM 3. *Let B be a closed R-invariant subspace of H^∞ with $B \supseteq \mathscr{R}_h$ such that either*
(i) *$B/\mathscr{R}_h$ is separable or*
(ii) *B is a countably generated $\mathscr{R}_h$ module.*
Then $B = \mathscr{R}_h$.

The proofs, especially of Theorem 1, are long, and we will give the details in a subsequent paper, giving here only an outline of the main steps in the proof of Theorem 1.

To begin with, we remark that M is isomorphic to PSL(2, $\boldsymbol{R}$), which is

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