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PONTRJAGIN CLASSES OF PL SHEAVES

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ABSTRACT. Over the category of PL manifolds there is a fibered category whose objects are certain equivalence classes $[\mathscr{F}]$ of "PL sheaves" $\mathscr{F}$, to which one assigns real characteristic classes as in [**2**] and [**3**]. In particular each PL manifold M possesses a distinguished (co)tangent object $[\mathscr{E}(M)]$ and a real Pontrjagin class $p([\mathscr{E}(M)])$. In this note we show that $p([\mathscr{E}(M)])$ is the image under $H^{4*}(M;Q) \to H^{4*}(M;R)$ of the Thom-Pontrjagin class of M.

The construction of [**2**] and [**3**] assigns total Chern classes $c([\mathscr{F}]) \in H^{2*}(M;R)$ to cosets $[\mathscr{F}]$ of complex PL sheaves $\mathscr{F}$ over a PL manifold M, and this assignment satisfies certain axioms. As in the classical case one defines the total Pontrjagin class $p([\mathscr{F}]) \in H^{4*}(M;R)$ of a coset $[\mathscr{F}]$ of real PL sheaves via complexification of $[\mathscr{F}]$, and here are the corresponding axioms:

(P_1) if $[\mathscr{F}]$ is a coset of real PL sheaves of "rank" m on a PL manifold M then the total Pontrjagin class $p([\mathscr{F}])$ is an element $1 + p_1([\mathscr{F}]) + \cdots + p_{[m/2]}([\mathscr{F}])$ of $H^*(M;R)$ with $p_i([\mathscr{F}]) \in H^{4i}(M;R)$;

(P_2) $p(\Xi^![\mathscr{F}]) = \Xi^* p([\mathscr{F}]) \in H^{4*}(N;R)$ for any PL map $\Xi : N \to M$;

(P_3) $p([\mathscr{F}] \oplus [\mathscr{G}]) = p([\mathscr{F}]) \cup p([\mathscr{G}])$ for any cosets $[\mathscr{F}]$ and $[\mathscr{G}]$ over M;

(P_4) if $[\mathscr{F}]$ contains a bona fide real vector bundle ξ over M (as in [**2**] or [**3**]) then $p([\mathscr{F}])$ is the classical total Pontrjagin class $p(\xi) \in H^{4*}(M;R)$.

LEMMA 1. *If a PL manifold M happens to admit a smooth structure with tangent bundle τ_M then $p([\mathscr{E}(M)]) = p(\tau_M)$.*

PROOF. One easily verifies as in [**2**] that $[\mathscr{E}(M)]$ contains τ_M; hence it suffices to apply (P_4).

As in the smooth case one uses axioms (P_1), (P_2), (P_3) and the multiplicative sequence corresponding to $z^{1/2}/(\tanh z^{1/2})$ to construct the Hirzebruch polynomial $l([\mathscr{F}]) \in H^{4*}(M; R)$ of the Pontrjagin class $p([\mathscr{F}])$,

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