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A COLLAPSE RESULT FOR THE EILENBERG-MOORE SPECTRAL SEQUENCE

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1. **Introduction.** In this note we outline a proof of the following result. Details will appear elsewhere.

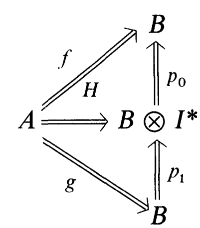
THEOREM. *Let $F \to E \to^p B$ be a fibration with E and B 1-connected. $H^*(E;Z_2)$ and $H^*(B;Z_2)$ polynomial algebras in finitely many variables. Suppose also that $Sq^{n-1}(y) = 0$ for all $y \in H^nE$. Then the Eilenberg-Moore spectral sequence with $E_2 = \mathrm{Tor}_{H^*(B;Z_2)}(H^*(E;Z_2), Z_2)$ and $E_r \Rightarrow H^*(F;Z_2)$ collapses.*

REMARKS. 1. The above applies very often to homogeneous spaces (take $G/H \to BH \to BG$).

2. For related results see e.g. [**1**], [**2**], [**3**], [**4**], [**5**], [**6**]. The main difference between our results and these earlier ones is that we do not have to impose conditions on the spaces, only on their cohomology.

3. Some evident generalizations (other coefficients, infinitely many generators for the polynomial algebras, fiber squares) are currently being worked out.

2. **Outline of proof.** Consider the category $\mathfrak{S}\mathfrak{H}\mathfrak{A}$ of differential graded algebras and shm maps, as defined in [7] (we impose some rather obvious extra normalization conditions). Write $f: A \Rightarrow B$. The category $\mathfrak{A}$ of differential graded algebras and multiplicative maps is the full subcategory determined by the condition $f_i = 0$ for $i > 1$. Let I^* be the normalized cochains on the semisimplicial 1-simplex. Define a homotopy (from f to g) to be an shm map $H: A \Rightarrow B \otimes I^*$ such that



commutes. Here $p_i: B \otimes I^* \Rightarrow B$ are the obvious multiplicative projections. There is then the associated homotopy category $\mathfrak{S}\mathfrak{H}\mathfrak{A}_h$. Given

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