BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 1, January 1973

# ON MINIMAL IMMERSIONS OF $S^2$ IN $S^{2m}$

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Communicated by S. S. Chern, August 9, 1972

1. **Introduction.** Let $x:S^2 \to S^{2m}(1)$ be a minimal immersion of the 2-sphere into the unit sphere of dimension $2m$. Following S. S. Chern [**4**] we associate to $x$ a certain holomorphic curve from $S^2$ with values in $CP^{2m}$ (the complex projective space of dimension $2m$) called the directrix curve of the minimal immersion. (Note that in the induced metric, $S^2$ acquires a conformal structure.) This curve is rational, and the unique condition it must satisfy is that of being totally isotropic, i.e. if $\xi$ is any of its local representations in homogeneous coordinates, then $\xi$ satisfies

$$(\xi, \xi) = (\xi', \xi') = \cdots = (\xi^{m-1}, \xi^{m-1}) = 0$$

where $( , )$ denotes the symmetric product in $C^{2m+1}$.

Chern proved, for the case $m = 2$, that the simple condition of total isotropy completely characterizes the set of directrix curves among all holomorphic ones from $S^2$ into $CP^{2m}$ if, instead of minimal immersions, we consider generalized minimal immersions. In the paper for which this is an announcement, we generalize this result and obtain further geometrical information about the corresponding minimal immersion. Complete proofs will appear elsewhere.

The first systematic study of this subject was made by E. Calabi who also associated, implicitly in (2) and explicitly in (3), a holomorphic curve $\eta$ to the minimal immersion $x$. It turns out that $\eta$ is the $(m - 1)$th associated curve of the directrix curve. This observation has allowed us to unify the approaches developed previously by Calabi and Chern.

2. **Definitions and preliminary remarks.** Let $x:S^2 \to S^{2m}$ be a differentiable map into the unit $2m$-sphere and let $z$ be a local isothermal parameter in $S^2$ relative to the induced metric. Set $\partial = \partial/\partial z$ and $\bar{\partial} = \partial/\partial \bar{z}$. Then we denote by

$$ds^2 = 2F|dz|^2, \quad F = (\partial x, \bar{\partial} x), \quad \text{the metric of } S^2,$$
$$\omega = iF dz \wedge d\bar{z}, \quad \text{the area form, and by}$$
$$K = -\partial\bar{\partial} \log(F)/F, \quad \text{the Gauss curvature.}$$

The minimality of $x$ is then equivalent to the equation

$$\partial\bar{\partial} x = -Fx.$$

*AMS (MOS) subject classifications* (1970). Primary 53B25, 32C25, 58D10; Secondary 14H45, 14H99, 14M15, 53C40.

[1] Research supported by Conselho Nacional de Pesquisas do Brazil and NSF Grant GP-34785X.