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THE COHOMOLOGY OF RESTRICTIONS OF THE $\bar{\partial}$ COMPLEX

BY RICARDO NIRENBERG[1]

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The local problem for complex vector fields (see for example Kohn [**3**]) can be summarized as follows: We are given a family $L_1, \ldots, L_m$ of vector fields on some neighborhood U of the origin in R^n:

$$L_i = \sum a_{ij} \frac{\partial}{\partial x_j}; \quad i = 1, \ldots, m,$$

where the a_{ij} are complex valued C^∞ functions on U. We assume that this family is closed under Lie brackets, i.e.

$$[L_i, L_j] = \sum d_{ij}^k L_k; \qquad i, j = 1, \ldots, m.$$

We then look at the equations

$$L_i(u) = f_i; \qquad i = 1, \ldots, m, \tag{1}$$

where the f_i are C^∞ functions on U, and try to give conditions on the L_i and the f_i so that a solution u should exist on maybe a smaller neighborhood of 0. We might also ask about the regularity properties of the solution u.

If we further assume that the L_i are linearly independent at each point of U, we can consider them as a basis for the sections of a vector bundle $\mathscr{L}$ on U, and we obtain a complex

$$C_0^\infty(V) \xrightarrow{D_0} \Gamma(\mathscr{L}^*, V) \xrightarrow{D_1} \Gamma(\mathscr{L}^* \wedge \mathscr{L}^*, V) \xrightarrow{D_2} \ldots, \qquad V \subset U.$$

Now equation (1) becomes $D_0(u) = f$ (see [**3**]).

For example, if M is a C^∞ submanifold of R^n, and d is the exterior derivative operator, the solution of the local problem is given by the Poincaré lemma and similarly, if M is a complex manifold, the solutions for the operators ∂ or $\bar{\partial}$ is given by the Dolbeault-Grothendieck lemma. In these two cases if M is compact we know that the cohomology spaces $H^i(M)$ ($i > 0$) are finite dimensional. A more difficult example is obtained if we take the restriction of the $\bar{\partial}$ (or the ∂) operator to a C^∞ real submanifold of a complex manifold (see [**2**], [**3**], [**4**] and [**6**]).

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