BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 1, January 1973

# A CHARACTERIZATION OF GROWTH IN LOCALLY COMPACT GROUPS

BY J. W. JENKINS[1]

Communicated by Calvin C. Moore, July 31, 1972

$G$ will denote throughout a separable, connected, locally compact group. Fix a left Haar measure on $G$ and for a measurable subset $A$ of $G$, let $|A|_G$ denote the measure of $A$. The purpose of this note is to announce results concerning the asymptotic behavior of $|U^n|_G$ where $U$ is a compact neighborhood of the identity $e$ in $G$, and to indicate some of the applications these results have for various areas. The following definitions are required:

DEFINITION 1. $G$ has *polynomial growth* if there is a polynomial $p$ such that for each compact neighborhood $U$ of $e$, there is a constant $C(U)$ so that

$$|U^n|_G \leqq C(U)p(n) \qquad (n = 1, 2, \ldots)$$

($U^n = \{u_1u_2, \ldots, u_n | u_i \in U, 1 \leqq i \leqq n\}$). $G$ has *exponential growth* if for each compact neighborhood $U$ of $e$ there is a $t > 1$ such that

$$|U^n|_G \geqq t^n \qquad (n = 1, 2, \ldots).$$

Note that since $G$ is connected, its "growth" will be determined by the behavior of $|U^n|_G$ for any one compact neighborhood $U$ of $e$.

For $a, b \in G$, let $[a, b]$ denote the subsemigroup of $G$ generated by $a$ and $b$, i.e.,

$$[a, b] = \{x_1x_2, \ldots, x_n | x_i \in \{a, b\}, 1 \leqq i \leqq n, n = 1, 2, \ldots\}.$$

$[a, b]$ is said to be free if $a[a, b] \cap b[a, b] = \varnothing$. A subset $S$ of $G$ is uniformly discrete if there is a neighborhood $U$ of $e$ in $G$ such that $sU \cap tU = \varnothing$ for $s, t \in S, s \neq t$.

DEFINITION 2. $G$ is *type NF* if there does not exist $a, b \in G$ such that $[a, b]$ is free and uniformly discrete.

Let $H$ be a connected Lie group with Lie algebra $\mathfrak{h}$, and let $g \to \mathrm{Ad}g$ be the canonical adjoint representation of $H$ on $\mathfrak{h}$. $H$ is said to be type $R$ if the eigenvalues of $\mathrm{Ad}g$ are of absolute value one for each $g \in H$.

Since $G$ is connected, there exists an arbitrarily small compact normal subgroup $K$ of $G$ such that $G/K$ is a Lie group.

DEFINITION 3. $G$ is *type R* if there exists a compact normal subgroup $K$

*AMS (MOS) subject classifications* (1969). Primary 22.20, 22.50, 28.75.

[1] This research was partially supported by NSF Grants GP-28925 and GP-7952X3.